VECTORS (Q 2, PAPER 2)

LESSON NO. 2: *i*, *j* VECTORS

2006

2 (a) $\vec{x} = -3\vec{i} + \vec{j}$. Express $(\vec{x}^{\perp})^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

SOLUTION

2 (a)

$\vec{r} = +x\vec{i} + y\vec{j} \Rightarrow \vec{r}^{\perp} = -y\vec{i} + x\vec{j}$ 7

$\vec{x} = -3\vec{i} + \vec{j} \Rightarrow \vec{x}^{\perp} = -\vec{i} - 3\vec{j}$

$\Rightarrow (\vec{x}^{\perp})^{\perp} = 3\vec{i} - \vec{j}$

2005

2 (b) $\vec{p} = 3\vec{i} + 4\vec{j}$. $\vec{q}$ is the unit vector in the direction of $\vec{p}$.

(i) Express $\vec{q}$ and $\vec{q}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Express $11\vec{i} - 2\vec{j}$ in the form $k\vec{q} + l\vec{q}^{\perp}$, where $k, l \in \mathbf{R}$.

SOLUTION

2 (b) (i)

Unit Vector: $\dfrac{\vec{r}}{|\vec{r}|} = \dfrac{x\vec{i} + y\vec{j}}{\sqrt{x^2 + y^2}}$ 6

$\vec{p} = 3\vec{i} + 4\vec{j} \Rightarrow \vec{q} = \dfrac{\vec{p}}{|\vec{p}|} = \dfrac{3\vec{i} + 4\vec{j}}{\sqrt{9+16}} = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}$

$\vec{r} = +x\vec{i} + y\vec{j} \Rightarrow \vec{r}^{\perp} = -y\vec{i} + x\vec{j}$ 7

$\vec{p}^{\perp} = -\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$

2 (b) (ii)

$11\vec{i} - 2\vec{j} = k\vec{q} + l\vec{q}^{\perp} \Rightarrow 11\vec{i} - 2\vec{j} = k(\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}) + l(-\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j})$

Equate the *i* and *j* parts: $\therefore 11 = \frac{3}{5}k - \frac{4}{5}l$ and $-2 = \frac{4}{5}k + \frac{3}{5}l$

Solve simultaneously for *k* and *l*:

$11 = \frac{3}{5}k - \frac{4}{5}l \Rightarrow 55 = 3k - 4l \;...(\mathbf{1})\,(\times 3)$

$-2 = \frac{4}{5}k + \frac{3}{5}l \Rightarrow -10 = 4k + 3l \;...(\mathbf{2})\,(\times 4)$

$\rightarrow$

$165 = 9k - 12l$

$-40 = 16k + 12l$

$125 = 25k \quad \Rightarrow k = 5$

Substitute this value of *k* into equation **1**: $\therefore 55 = 3(5) - 4l \Rightarrow 40 = -4l \Rightarrow l = -10$

Ans: $5\vec{q} - 10\vec{q}^{\perp}$

2004

2 (a) $\vec{r} = 12\vec{i} - 35\vec{j}$. Find the unit vector in the direction of $\vec{r}$.

SOLUTION

2 (a)

$\vec{r} = 12\vec{i} - 35\vec{j}$

Unit Vector: $\dfrac{\vec{r}}{|\vec{r}|} = \dfrac{x\vec{i} + y\vec{j}}{\sqrt{x^2 + y^2}}$ 6

Unit vector $= \dfrac{\vec{r}}{|\vec{r}|} = \dfrac{12\vec{i} - 35\vec{j}}{\sqrt{12^2 + 35^2}} = \dfrac{12\vec{i} - 35\vec{j}}{37} = \frac{12}{37}\vec{i} - \frac{35}{37}\vec{j}$

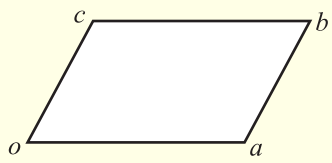
2003

2 (a) *oabc* is a parallelogram where *o* is the origin, $\vec{a} = 3\vec{i} - \vec{j}$ and $\vec{b} = 4\vec{i} + 3\vec{j}$. Express $\vec{c}$ in terms of $\vec{i}$ and $\vec{j}$.

SOLUTION

2 (a)

$\vec{c} = \overrightarrow{oc} = \overrightarrow{ab} = \vec{b} - \vec{a} = (4\vec{i} + 3\vec{j}) - (3\vec{i} - \vec{j}) = \vec{i} + 4\vec{j}$



2002

2 (a) $\vec{s} = 4\vec{i} - 3\vec{j}$ and $\vec{t} = 2\vec{i} - 5\vec{j}$. Find $|\overrightarrow{st}|$.

SOLUTION

2 (a)

$|\overrightarrow{st}| = |\vec{t} - \vec{s}| = |(2\vec{i} - 5\vec{j}) - (4\vec{i} - 3\vec{j})| = |-2\vec{i} - 2\vec{j}|$

$= \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

$\overrightarrow{ab} = \vec{b} - \vec{a}$ 1

$\vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2}$ 5

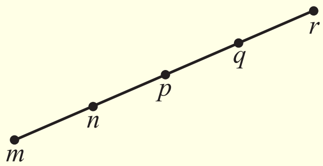
2001

2 (b) [*mr*] is divided into four line segments of equal length by the points *n*, *p* and *q*.

Given that $\vec{m} = -2\vec{i} + 3\vec{j}$ and $\vec{q} = 7\vec{i} - 9\vec{j}$, express

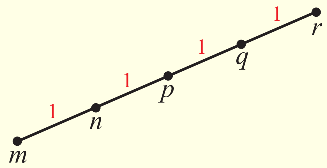
(i) $\vec{p}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.



SOLUTION

2 (b) (i)



$$\vec{p} = \frac{\vec{m} + 2\vec{q}}{3} = \frac{(-2\vec{i} + 3\vec{j}) + 2(7\vec{i} - 9\vec{j})}{3}$$

$$= \frac{12\vec{i} - 15\vec{j}}{3} = 4\vec{i} - 5\vec{j}$$

2 (b) (ii)

$$\vec{q} = \frac{3\vec{r} + \vec{m}}{4} \Rightarrow \vec{r} = \frac{4\vec{q} - m}{3} = \frac{4(7\vec{i} - 9\vec{j}) - (-2\vec{i} + 3\vec{j})}{3}$$

$$= \frac{30\vec{i} - 39\vec{j}}{3} = 10\vec{i} - 13\vec{j}$$