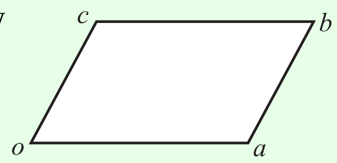
VECTORS (Q 2, PAPER 2)

LESSON NO. 1: BASIC VECTORS

2005

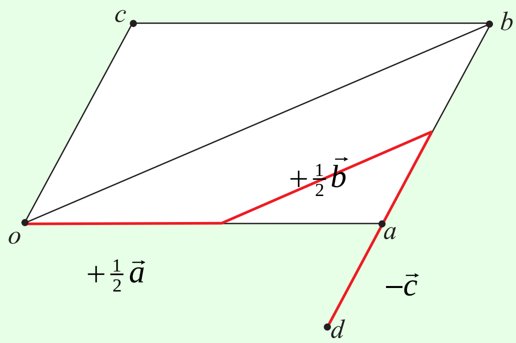
2 (a) Copy the parallelogram oabc in your answerbook. Show your work, construct the point d such that

$\vec{d} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} - \vec{c}$, where o is the origin.



SOLUTION

2 (a)



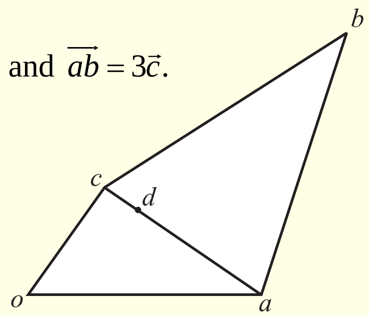
2004

2 (b) $oabc$ is a quadrilateral, where o is the origin. $\overrightarrow{ad} = 3\overrightarrow{dc}$ and $\overrightarrow{ab} = 3\vec{c}$.

(i) Express $\vec{d}$ in terms of $\vec{a}$ and $\vec{c}$.

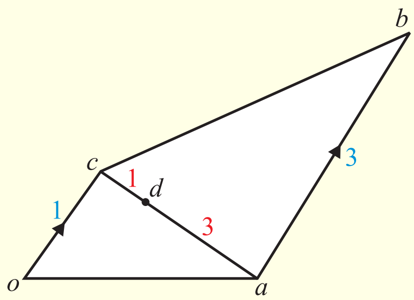
(ii) Express $\overrightarrow{db}$ in terms of $\vec{a}$ and $\vec{c}$.

(iii) Show that o, d and b are collinear.



SOLUTION

2 (b) From the information provided, d divides [ac] in the ratio 3:1 and [ab] is three times longer than [oc] and parallel to it.



2 (b) (i)

$$\vec{c} = \frac{m\vec{b} + n\vec{a}}{m+n}$$ **2**

$$\vec{d} = \frac{1\vec{a} + 3\vec{c}}{4} = \frac{1}{4}\vec{a} + \frac{3}{4}\vec{c}$$

2 (b) (ii)

$$\overrightarrow{ab} = \vec{b} - \vec{a}$$ **1**

$$\overrightarrow{ab} = 3\vec{c} \Rightarrow \vec{b} - \vec{a} = 3\vec{c} \Rightarrow \vec{b} = \vec{a} + 3\vec{c}$$

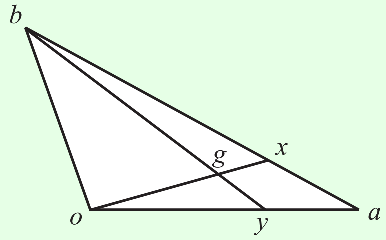
$$\overrightarrow{db} = \vec{b} - \vec{d} = \vec{a} + 3\vec{c} - \frac{1}{4}\vec{a} - \frac{3}{4}\vec{c} = \frac{3}{4}\vec{a} + \frac{9}{4}\vec{c}$$

2003

2 (c) oab is a triangle where o is the origin.

(i) x is a point on $[ab]$ such that $|ax|:|xb| = 1:3$.

Express $\vec{x}$ in terms of $\vec{a}$ and $\vec{b}$.



(ii) y is a point on $[oa]$ such that $|oy|:|ya| = 2:1$.

Express $\overrightarrow{by}$ in terms of $\vec{a}$ and $\vec{b}$.

(iii) $[ox]$ and $[by]$ intersect at g. Given that

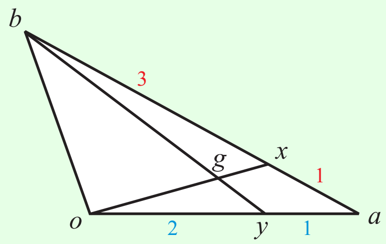
$\vec{g} = m\vec{x}$ and $\overrightarrow{bg} = n\overrightarrow{by}$ where $m, n \in \mathbf{R}$,

find the value of m and the value of n.

SOLUTION

2 (c) (i)

$$\vec{c} = \frac{m\vec{b} + n\vec{a}}{m+n} \text{ .... } \textcircled{2}$$



$$\vec{x} = \frac{3\vec{a} + \vec{b}}{4} = \tfrac{3}{4}\vec{a} + \tfrac{1}{4}\vec{b}$$

2 (c) (ii)

$$\vec{y} = \tfrac{2}{3}\vec{a}$$

$$\overrightarrow{by} = \vec{y} - \vec{b} = \tfrac{2}{3}\vec{a} - \vec{b}$$

2 (c) (iii)

$$\vec{g} = m\vec{x} = \tfrac{3}{4}m\vec{a} + \tfrac{1}{4}m\vec{b} \text{ ....}(\mathbf{1})$$

$$\overrightarrow{bg} = n\overrightarrow{by} \Rightarrow \vec{g} - \vec{b} = \tfrac{2}{3}n\vec{a} - n\vec{b} \Rightarrow \vec{g} = \tfrac{2}{3}n\vec{a} + (1-n)\vec{b} \text{ ....}(\mathbf{2})$$

Equating (**1**) and (**2**) $\Rightarrow \tfrac{3}{4}m\vec{a} + \tfrac{1}{4}m\vec{b} = \tfrac{2}{3}n\vec{a} + (1-n)\vec{b}$

Equate the $\vec{a}$ and $\vec{b}$ parts $\Rightarrow \tfrac{3}{4}m = \tfrac{2}{3}n$ ...(**1**) and $\tfrac{1}{4}m = (1-n)$ ...(**2**)

From equation (**1**): $\Rightarrow 9m = 8n \Rightarrow n = \tfrac{9}{8}m$

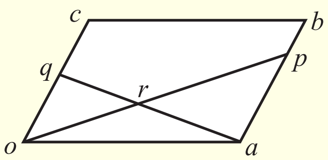
Substitute this value of n into equation (**2**): $\tfrac{1}{4}m = (1-n) \Rightarrow \tfrac{1}{4}m = 1 - \tfrac{9}{8}m \Rightarrow \tfrac{11}{8}m = 1 \Rightarrow m = \tfrac{8}{11}$

$\Rightarrow n = \tfrac{9}{8}(\tfrac{8}{11}) = \tfrac{9}{11}$

2002

2 (b) *oabc* is a parallelogram, where *o* is the origin. $p \in [ab]$ such that $|ap|:|pb| = 3:1$.
q is the midpoint of [*oc*].

(i) Using equiangular triangles, or otherwise, find the ratio $|or|:|rp|$.



(ii) Express $\vec{p}$, and hence $\vec{r}$, in terms of $\vec{a}$ and $\vec{b}$.

Solution

2 (b) (i)

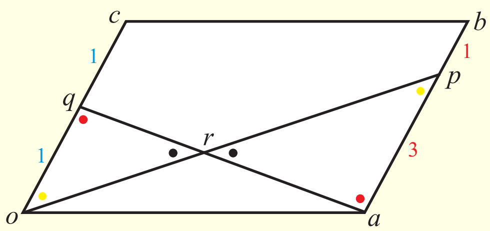
Consider triangles $\Delta\, qro$ and $\Delta\, pra$.

$\angle qro = \angle pra$ (Vertically opposite angles)

$\angle rqo = \angle par$ (Alternate angles)

$\angle roq = \angle apr$ (Alternate angles)

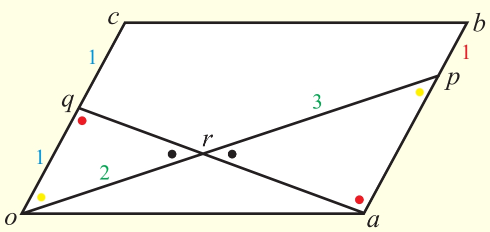
Therefore, triangles $\Delta\, qro$ and $\Delta\, pra$ are equi-angular. This means the ratio of their corresponding sides are equal.



$$\therefore \frac{|or|}{|rp|} = \frac{|oq|}{|ap|} = \frac{\frac{1}{2}|oc|}{\frac{3}{4}|ab|} = \frac{2|oc|}{3|oc|} = \frac{2}{3} \Rightarrow |or|:|rp| = 2:3$$

2 (b) (ii)

$$\vec{c} = \frac{m\vec{b} + n\vec{a}}{m+n}$$ 2



$$\vec{p} = \frac{\vec{a} + 3\vec{b}}{4} = \tfrac{1}{4}\vec{a} + \tfrac{3}{4}\vec{b}$$

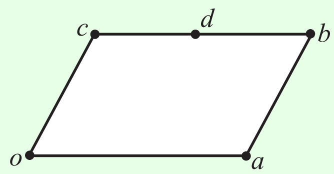
$$\vec{r} = \tfrac{2}{5}\vec{p} = \tfrac{2}{5}(\tfrac{1}{4}\vec{a} + \tfrac{3}{4}\vec{b}) = \tfrac{1}{10}\vec{a} + \tfrac{3}{10}\vec{b}$$

2001

2 (a) *oabc* is a parallelogram where *o* is the origin. *d* is the midpoint of [*cb*].

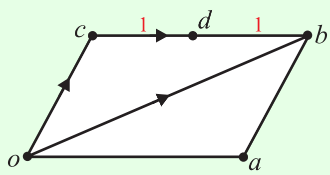
(i) Express $\vec{b}$ in terms of $\vec{a}$ and $\vec{c}$.

(ii) Express $\vec{d}$ in terms of $\vec{a}$ and $\vec{c}$.



Solution

2 (a) (i)



$\vec{b} = \vec{c} + \overrightarrow{cb}$

However, $\vec{a} = \overrightarrow{cb} \Rightarrow \vec{b} = \vec{a} + \vec{c}$