VECTORS (Q 2, PAPER 2)

2010

2 (a) A, B and C are points and O is the origin.

$\vec{a} = 2\vec{i} + 3\vec{j}$, $\vec{b} = -3\vec{i} - 6\vec{j}$, and $\overrightarrow{AC} = \overrightarrow{OB}$.

Express $\vec{c}$ in terms of $\vec{i}$ and $\vec{j}$.

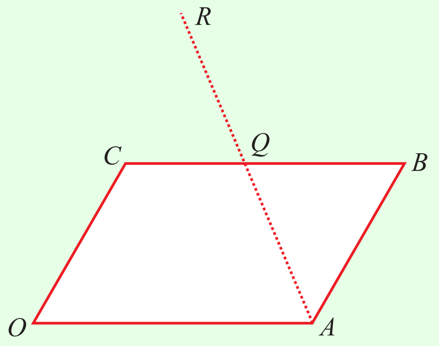
(b) $\vec{u} = 2\vec{i} + \vec{j}$ and $\vec{v} = -\vec{i} + k\vec{j}$ where $k \in \mathbf{R}$.

(i) Express $|\vec{v}|$ and $\vec{u}.\vec{v}$ in terms of k.

(ii) Given that $\cos\theta = -\frac{1}{\sqrt{2}}$, where θ is the angle between $\vec{u}$ and $\vec{v}$,

find the two possible values of k.

(c) $OABC$ is a parallelogram, where O is the origin.
Q is the midpoint of $[BC]$.
$[AQ]$ is extended to R such that $|AQ| = |QR|$.



(i) Express $\vec{q}$ in terms of $\vec{a}$ and $\vec{c}$.

(ii) Express $\overrightarrow{AQ}$ in terms of $\vec{a}$ and $\vec{c}$.

(iii) Show that the points O, C and R are collinear.

SOLUTION

2 (a)

$\overrightarrow{AC} = \overrightarrow{OB}$

$\vec{c} - \vec{a} = b$

$\vec{c} = \vec{a} + \vec{b} = (2\vec{i} + 3\vec{j}) + (-3\vec{i} - 6\vec{j})$

$= -\vec{i} - 3\vec{j}$

$\overrightarrow{ab} = \vec{b} - \vec{a}$

2 (b) (i)

$\vec{u} = 2\vec{i} + \vec{j}$

$\vec{v} = -\vec{i} + k\vec{j}$

$|\vec{v}| = \sqrt{(-1)^2 + k^2} = \sqrt{1 + k^2}$

$\vec{u}.\vec{v} = (2\vec{i} + \vec{j}).(-\vec{i} + k\vec{j}) = -2 + k$

$\vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2}$

DOT PRODUCT: Just multiply the $\vec{i}$ coefficients and multiply the $\vec{j}$ coefficients.

2 (b) (ii)

$\vec{u}.\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

$\overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos\theta$

$-2+k = |2\vec{i}+\vec{j}|\sqrt{1+k^2} \times -\frac{1}{\sqrt{2}}$

$-2+k = \sqrt{5}\sqrt{1+k^2} \times -\frac{1}{\sqrt{2}}$

$\vec{r} = x\vec{i}+y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2+y^2}$

$(-2+k)^2 = \left(\sqrt{5}\sqrt{1+k^2} \times -\frac{1}{\sqrt{2}}\right)^2$

$4-4k+k^2 = \frac{5}{2}(1+k^2)$

$8-8k+2k^2 = 5+5k^2$

$0 = 3k^2+8k-3$

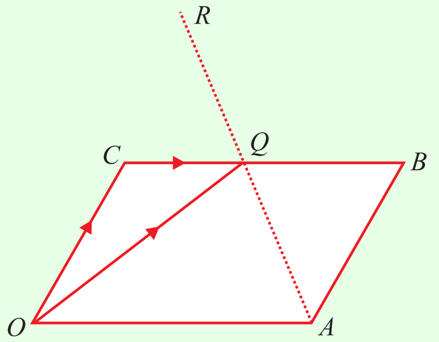
$0 = (3k-1)(k+3)$

$\therefore k = -3, \frac{1}{3}$

2 (c) (i)

$\overrightarrow{OQ} = \overrightarrow{OC} + \overrightarrow{CQ}$

$\vec{q} = \vec{c} + \frac{1}{2}\vec{a}$



2 (c) (ii)

$\overrightarrow{AQ} = \vec{q} - \vec{a}$

$\overrightarrow{ab} = \vec{b} - \vec{a}$

$= \vec{c} + \frac{1}{2}\vec{a} - \vec{a}$

$= \vec{c} - \frac{1}{2}\vec{a}$

2 (c) (iii)

$\vec{r} = \vec{a} + \overrightarrow{AR}$

$\vec{r} = \vec{a} + 2\overrightarrow{AQ}$

$\vec{r} = \vec{a} + 2(\vec{c} - \frac{1}{2}\vec{a})$

$\vec{r} = \vec{a} + 2\vec{c} - \vec{a}$

$\vec{r} = 2\vec{c}$

a, c, b collinear $\Rightarrow \overrightarrow{ac} = k\overrightarrow{ab}$

Therefore, O, C and R are collinear.