VECTORS (Q 2, PAPER 2)

2009

2 (a) If $\vec{a} = 2\vec{i} + \vec{j}$, $\vec{b} = -\vec{i} + 5\vec{j}$, find the unit vector in the direction of $\overrightarrow{ab}$.

(b) In the triangle abc, p is a point on the side $[bc]$.
The point q lies outside the triangle such that $\overrightarrow{pq} = \overrightarrow{pb} + \overrightarrow{pc} - \overrightarrow{pa}$.

(i) Express $\vec{q}$ in terms of $\vec{a}$, $\vec{b}$ and $\vec{c}$.

(ii) Hence show that $abqc$ is a parallelogram.

(c) (i) $\vec{p} = 12\vec{i} + 5\vec{j}$ and $\vec{q} = 3\vec{i} + 4\vec{j}$.
Find the value of the scalar k such that $k\left|\vec{p}^{\perp} - \vec{q}\right| = \left|\vec{p}^{\perp}\right| - |\vec{q}|$.

(ii) Prove that for all vectors $\vec{r}$ and $\vec{s}$
$(\vec{r} - \vec{s})^{\perp} = \vec{r}^{\perp} - \vec{s}^{\perp}$.

SOLUTION

2 (a)

$\vec{a} = 2\vec{i} + \vec{j}$

$\vec{b} = -\vec{i} + 5\vec{j}$

$$\overrightarrow{ab} = \vec{b} - \vec{a} = 2\vec{i} + \vec{j} - (-\vec{i} + 5\vec{j})$$
$$= 2\vec{i} + \vec{j} + \vec{i} - 5\vec{j}$$
$$= 3\vec{i} - 4\vec{j}$$

$\overrightarrow{ab} = \vec{b} - \vec{a}$

$$\frac{\overrightarrow{ab}}{\left|\overrightarrow{ab}\right|} = \frac{3\vec{i} - 4\vec{j}}{\left|3\vec{i} - 4\vec{j}\right|} = \frac{3\vec{i} - 4\vec{j}}{\sqrt{3^2 + (-4)^2}}$$
$$= \frac{3\vec{i} - 4\vec{j}}{\sqrt{25}} = \frac{3\vec{i} - 4\vec{j}}{5}$$
$$= \tfrac{3}{5}\vec{i} - \tfrac{4}{5}\vec{j}$$

Unit Vector: $\dfrac{\vec{r}}{|\vec{r}\,|} = \dfrac{x\vec{i} + y\vec{j}}{\sqrt{x^2 + y^2}}$

$\vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}\,| = \sqrt{x^2 + y^2}$

2 (b) (i)

$\overrightarrow{pq} = \overrightarrow{pb} + \overrightarrow{pc} - \overrightarrow{pa}$

$\vec{q} - \vec{p} = \vec{b} - \vec{p} + \vec{c} - \vec{p} - (\vec{a} - \vec{p})$

$\vec{q} - \vec{p} = \vec{b} - \vec{p} + \vec{c} - \vec{p} - \vec{a} + \vec{p}$

$\vec{q} = \vec{b} + \vec{c} - \vec{a}$

2 (b) (ii)

$\vec{q} = \vec{b} + \vec{c} - \vec{a}$

$\vec{q} - \vec{b} = \vec{c} - \vec{a}$

$\overrightarrow{bq} = \overrightarrow{ac}$

Side bq is parallel and the same size as side ac.
Therefore, $abqc$ is a parallelogram.

2 (c) (i)

$\vec{p} = 12\vec{i} + 5\vec{j} \Rightarrow \vec{p}^{\perp} = -5\vec{i} + 12\vec{j}$

$\vec{r} = +x\vec{i} + y\vec{j} \Rightarrow \vec{r}^{\perp} = -y\vec{i} + x\vec{j}$

$\vec{q} = 3\vec{i} + 4\vec{j} \Rightarrow \vec{q}^{\perp} = -4\vec{i} + 3\vec{j}$

$k\left|\vec{p}^{\perp} - \vec{q}\right| = \left|\vec{p}^{\perp}\right| - \left|\vec{q}\right|$

$k\left|-5\vec{i} + 12\vec{j} - 3\vec{i} - 4\vec{j}\right| = \left|-5\vec{i} + 12\vec{j}\right| - \left|3\vec{i} + 4\vec{j}\right|$

$k\left|-8\vec{i} + 8\vec{j}\right| = \left|-5\vec{i} + 12\vec{j}\right| - \left|3\vec{i} + 4\vec{j}\right|$

$k\sqrt{(-8)^2 + 8^2} = \sqrt{(-5)^2 + 12^2} - \sqrt{3^2 + 4^2}$

$\vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2}$

$k\sqrt{128} = \sqrt{169} - \sqrt{25}$

$8k\sqrt{2} = 13 - 5$

$8k\sqrt{2} = 8$

$k = \dfrac{1}{\sqrt{2}}$

2 (c) (ii)

Let $\vec{r} = a\vec{i} + b\vec{j}$, $\vec{s} = c\vec{i} + d\vec{j}$

LHS	**RHS**
$(\vec{r} - \vec{s})^{\perp}$	$\vec{r}^{\perp} - \vec{s}^{\perp}$
$= (a\vec{i} + b\vec{j} - c\vec{i} - d\vec{j})^{\perp}$	$= (a\vec{i} + b\vec{j})^{\perp} - (c\vec{i} + d\vec{j})^{\perp}$
$= ((a-c)\vec{i} + (b-d)\vec{j})^{\perp}$	$= (-b\vec{i} + a\vec{j}) - (-d\vec{i} + c\vec{j})$
$= (d-b)\vec{i} + (a-c)\vec{j}$	$= -b\vec{i} + a\vec{j} + d\vec{i} - c\vec{j}$
	$= (d-b)\vec{i} + (a-c)\vec{j}$