VECTORS (Q 2, PAPER 2)

2008

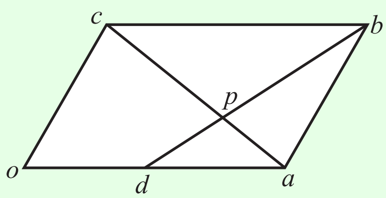
2 (a) Given that $|10\vec{i}+k\vec{j}|=|11\vec{i}-2\vec{j}|$, find the two possible values of $k \in \mathbf{R}$.

(b) $\vec{x}=-\vec{i}+3\vec{j}$, $\vec{y}=4\vec{i}-2\vec{j}$ and $\vec{z}=\vec{x}-t\vec{y}$, where $t \in \mathbf{R}$.

(i) Given that $\vec{x} \perp \vec{z}$, calculate the value of t.

(ii) Find the measure of $\angle xoy$, where o is the origin.

(c) $oabc$ is a parallelogram, where o is the origin.
d is the midpoint of $[oa]$ and $[db]$ cuts the diagonal $[ac]$ at p.



(i) Given that $\overrightarrow{ap}=k\overrightarrow{ac}$, where $k \in \mathbf{R}$,
express $\vec{p}$ in terms of $\vec{a}$, $\vec{c}$ and k.

(ii) Given that $\overrightarrow{bp}=l\overrightarrow{bd}$, where $l \in \mathbf{R}$, express $\vec{p}$ in terms of $\vec{a}$, $\vec{c}$ and l.

(iii) Hence find the value of k and the value of l.

SOLUTION

2 (a)

$|10\vec{i}+k\vec{j}|=|11\vec{i}-2\vec{j}|$

$\vec{r}=x\vec{i}+y\vec{j} \Rightarrow |\vec{r}|=\sqrt{x^2+y^2}$ 5

$\Rightarrow \sqrt{10^2+k^2}=\sqrt{11^2+(-2)^2}$

$\Rightarrow 100+k^2=125$

$\Rightarrow k^2=25$

$\therefore k=\pm 5$

2 (b) (i)

$\vec{x}=-\vec{i}+3\vec{j}$

$\vec{y}=4\vec{i}-2\vec{j}$

$\vec{z}=\vec{x}-t\vec{y}=(-\vec{i}+3\vec{j})-t(4\vec{i}-2\vec{j})$

$\Rightarrow \vec{z}=-\vec{i}+3\vec{j}-4t\vec{i}+2t\vec{j}$

$\therefore \vec{z}=(-1-4t)\vec{i}+(3+2t)\vec{j}$

If two vectors are perpendicular their dot product is zero.
If their dot product is zero, the vectors are perpendicular.

$\vec{i}.\vec{j}=\vec{j}.\vec{i}=0$ 9

$\vec{i}.\vec{i}=\vec{j}.\vec{j}=|\vec{i}|^2=|\vec{j}|^2=1$ 10

$\vec{x} \perp \vec{z} \Rightarrow \vec{x}.\vec{z}=0$

$\Rightarrow (-\vec{i}+3\vec{j}).((-1-4t)\vec{i}+(3+2t)\vec{j})$

$\Rightarrow 1+4t+9+6t=0$

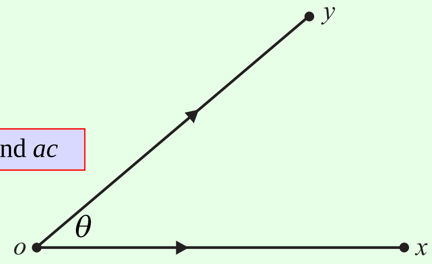
$\Rightarrow 10t=-10$

$\therefore t=-1$

When you dot two vectors just multiply the $\vec{i}$ coefficients and multiply the $\vec{j}$ coefficients.

2 (b) (ii)

$\vec{ab}.\vec{ac} = |\vec{ab}||\vec{ac}|\cos\theta$ **8**



Remember it as:

ab dot *ac* = Length [*ab*] × Length [*ac*] × cos of angle between *ab* and *ac*

$\vec{x}.\vec{y} = |\vec{x}||\vec{y}|\cos\theta \Rightarrow \cos\theta = \dfrac{\vec{x}.\vec{y}}{|\vec{x}||\vec{y}|}$

$\Rightarrow \cos\theta = \dfrac{(-\vec{i}+3\vec{j}).(4\vec{i}-2\vec{j})}{|-\vec{i}+3\vec{j}||4\vec{i}-2\vec{j}|}$

$\Rightarrow \cos\theta = \dfrac{-4-6}{\sqrt{1+9}\sqrt{16+4}}$

$\Rightarrow \cos\theta = \dfrac{-10}{\sqrt{10}\sqrt{20}} = -\dfrac{1}{\sqrt{2}}$ [The negative sign means the angle is obtuse.]

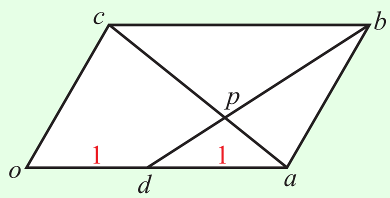
$\therefore \theta = 135^\circ$

2 (c) (i)

$\vec{ap} = k\vec{ac} \Rightarrow \vec{p} - \vec{a} = k(\vec{c} - \vec{a})$

$\Rightarrow \vec{p} = \vec{a} + k\vec{c} - k\vec{a}$

$\therefore \vec{p} = (1-k)\vec{a} + k\vec{c}$



$\vec{ab} = \vec{b} - \vec{a}$ **1**

2 (c) (ii)

$\vec{bp} = l\vec{bd} \Rightarrow \vec{p} - \vec{b} = l(\vec{d} - \vec{b})$

$\Rightarrow \vec{p} = \vec{b} + l(\vec{d} - \vec{b})$

Express vectors *b* and *d* in terms of *a* and *c*.

$\vec{b} = \vec{a} + \vec{c}$

$\vec{d} = \frac{1}{2}\vec{a}$

$\therefore \vec{p} = \vec{a} + \vec{c} + l(\frac{1}{2}\vec{a} - \vec{a} - \vec{c})$

$\Rightarrow \vec{p} = \vec{a} + \vec{c} + l(-\frac{1}{2}\vec{a} - \vec{c})$

$\Rightarrow \vec{p} = \vec{a} + \vec{c} - \frac{1}{2}l\vec{a} - l\vec{c}$

$\therefore \vec{p} = (1 - \frac{1}{2}l)\vec{a} + (1-l)\vec{c}$

2 (c) (iii)

Equate the *p* vectors from parts **(i)** and **(ii)**.

$\therefore (1-k)\vec{a} + k\vec{c} = (1 - \frac{1}{2}l)\vec{a} + (1-l)\vec{c}$

$\Rightarrow 1 - k = 1 - \frac{1}{2}l$ and $k = 1 - l$ [Equate the coefficients.]

$\therefore 1 - (1-l) = 1 - \frac{1}{2}l$

$\Rightarrow l = 1 - \frac{1}{2}l \Rightarrow \frac{3}{2}l = 1$

$\therefore l = \frac{2}{3}$

$k = 1 - l = 1 - \frac{2}{3}$

$\therefore k = \frac{1}{3}$