VECTORS (Q 2, PAPER 2)

2000

2 (a) $\vec{v} = t\vec{i} - 8\vec{j}$, where $t \in \mathbf{R}$.

Find the two possible values of t for which $|\vec{v}| = 17$.

2 (b) $\vec{a} = 2\vec{i} + (2k+3)\vec{j}$ and $\vec{b} = k^2\vec{i} + 6\vec{j}$, where $k \in \mathbf{Z}$.

$\vec{a}$ is perpendicular to $\vec{b}$.

(i) Find the value of k.

(ii) Using your value of k, write $\vec{a} + \vec{b}$ in terms of $\vec{i}$ and $\vec{j}$.

(iii) Hence, find the measure of the angle between $\vec{a}$ and $\vec{a} + \vec{b}$ correct to the nearest degree.

2 (c) (i) $\vec{p} + \vec{q} = 5\vec{i} - 5\vec{j}$ and $\overrightarrow{pq} = 3\vec{i} + \vec{j}$.

Express $\vec{p}$ and $\vec{q}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Given that $\vec{r} = \left(\dfrac{\vec{p}.\vec{q}}{\vec{q}.\vec{q}}\right)\vec{q}$, express $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

(iii) Given that $\vec{s} = \frac{7}{2}\vec{i} + m\vec{j}$, $m \in \mathbf{Q}$, find the value of m for which the origin, r and s are collinear.

SOLUTION

2 (a)

$\vec{v} = t\vec{i} - 8\vec{j} \Rightarrow |\vec{v}| = \sqrt{t^2 + (-8)^2} = 17$

$\vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2}$ 5

$\Rightarrow \sqrt{t^2 + 64} = 17$

$\Rightarrow t^2 + 64 = 289$

$\Rightarrow t^2 = 225 \Rightarrow t = \sqrt{225}$

$\therefore t = \pm 15$

2 (b) (i)

If two vectors are perpendicular their dot product is zero. If their dot product is zero, the vectors are perpendicular.

When you dot two vectors, just multiply the $\vec{i}$ coefficients and multiply the $\vec{j}$ coefficients.

$\vec{a}.\vec{b} = 2k^2 + 6(2k+3) = 0$

$\Rightarrow 2k^2 + 12k + 18 = 0$

$\Rightarrow k^2 + 6k + 9 = 0$

$\Rightarrow (k+3)(k+3) = 0$

$\therefore k = -3$

2 (b) (ii)

$\vec{a} = 2\vec{i} - 3\vec{j},\ \vec{b} = 9\vec{i} + 6\vec{j}$

$\therefore \vec{a} + \vec{b} = 11\vec{i} + 3\vec{j}$

2 (b) (iii)

$\vec{a}.(\vec{a} + \vec{b}) = |\vec{a}||\vec{a} + \vec{b}|\cos\theta$ $\overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos\theta$ **8**

$\Rightarrow (2\vec{i} - 3\vec{j}).(11\vec{i} + 3\vec{j}) = \sqrt{2^2 + (-3)^2}\sqrt{11^2 + 3^3}\cos\theta$

$\Rightarrow 22 - 9 = \sqrt{13}\sqrt{130}\cos\theta$

$\Rightarrow 13 = \sqrt{13}\sqrt{130}\cos\theta$

$\Rightarrow \cos\theta = \dfrac{13}{\sqrt{13}\sqrt{130}} = \dfrac{1}{\sqrt{10}}$

$\therefore \theta = \cos^{-1}\left(\dfrac{1}{\sqrt{10}}\right) = 72^o$

2 (c) (i)

$\vec{p} + \vec{q} = 5\vec{i} - 5\vec{j}$....**(1)**

$\overrightarrow{pq} = \vec{q} - \vec{p} = 3\vec{i} + \vec{j}$...**(2)** $\overrightarrow{ab} = \vec{b} - \vec{a}$ **1**

$(\mathbf{1}) + (\mathbf{2}): 2\vec{q} = 8\vec{i} - 4\vec{j} \Rightarrow \vec{q} = 4\vec{i} - 2\vec{j}$

Substitute this value of vector q into Eqn. (**1**): $\therefore \vec{p} + 4\vec{i} - 2\vec{j} = 5\vec{i} - 5\vec{j} \Rightarrow \vec{p} = \vec{i} - 3\vec{j}$

2 (c) (ii)

$\vec{r} = \left(\dfrac{\vec{p}.\vec{q}}{\vec{q}.\vec{q}}\right)\vec{q} = \left(\dfrac{(\vec{i} - 3\vec{j}).(4\vec{i} - 2\vec{j})}{(4\vec{i} - 2\vec{j}).(4\vec{i} - 2\vec{j})}\right)(4\vec{i} - 2\vec{j})$

$\Rightarrow \vec{r} = \left(\dfrac{4+6}{16+4}\right)(4\vec{i} - 2\vec{j})$

$\Rightarrow \vec{r} = \left(\dfrac{10}{20}\right)(4\vec{i} - 2\vec{j})$

$\therefore \vec{r} = \tfrac{1}{2}(4\vec{i} - 2\vec{j}) = 2\vec{i} - \vec{j}$

2 (c) (iii)

COLLINEARITY
Three points are collinear if a vector formed from any two is a scalar multiplied by a vector formed from any other 2 points.

a, c, b collinear $\Rightarrow \overrightarrow{ac} = k\overrightarrow{ab}$

o, s and r collinear: $\overrightarrow{os} = k\overrightarrow{or} \Rightarrow \vec{s} = k\vec{r}$

$\Rightarrow \tfrac{7}{2}\vec{i} + m\vec{j} = k(2\vec{i} - \vec{j})$

$\Rightarrow \tfrac{7}{2}\vec{i} + m\vec{j} = 2k\vec{i} - k\vec{j}$

$\therefore \tfrac{7}{2}\vec{i} = 2k\vec{i} \Rightarrow \tfrac{7}{2} = 2k \Rightarrow k = \tfrac{7}{4}$

$\therefore m\vec{j} = -k\vec{j} \Rightarrow m = -k \Rightarrow m = -\tfrac{7}{4}$