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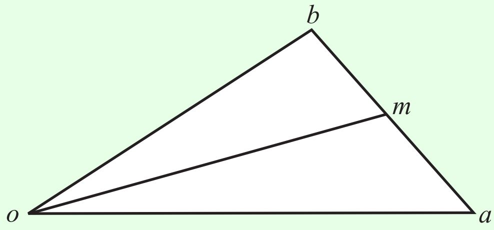
2 (a) $\vec{p} = 3\vec{i} + 2\vec{j}$, $\vec{q} = 5\vec{i} - 6\vec{j}$ and $\vec{p} = \overrightarrow{rq}$.

Express $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

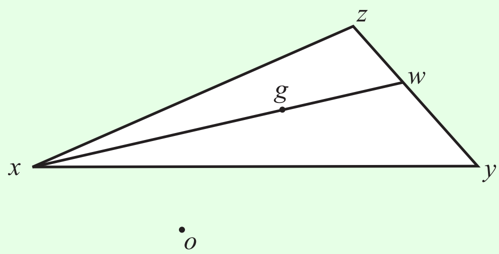
(b) (i) *oab* is a triangle where *o* is the origin. *m* is the midpoint of [*ab*].

Express $\vec{m}$ in terms of $\vec{a}$ and $\vec{b}$.



(ii) In the triangle *xyz*, *w* is the midpoint of [*yz*], *g* is a point in [*xw*] such that $|xg| = \frac{2}{3}|xw|$ and *o* is the origin.

Express $\vec{g}$ in terms of $\vec{x}$, $\vec{y}$ and $\vec{z}$.



(c) (i) Show for all vectors $\vec{r}$ and $\vec{s}$ that $\vec{r}.\vec{s}^{\perp} = -\vec{r}^{\perp}.\vec{s}$.

(ii) *a*, *b* and *c* are distinct points. If $\overrightarrow{ad} = t\left(\frac{\overrightarrow{ab}^{\perp}}{|\overrightarrow{ab}|} - \frac{\overrightarrow{ac}^{\perp}}{|\overrightarrow{ac}|}\right)$, where $t \in \mathbf{R}$ and

$t \neq 0$, show that $|\angle bad| = |\angle cad|$.

SOLUTION

2 (a)

$\vec{p} = \overrightarrow{rq} \Rightarrow \vec{p} = \vec{q} - \vec{r}$

$\overrightarrow{ab} = \vec{b} - \vec{a}$ 1

$\therefore \vec{r} = \vec{q} - \vec{p} = (5\vec{i} - 6\vec{j}) - (3\vec{i} + 2\vec{j})$

$\Rightarrow \vec{r} = 5\vec{i} - 6\vec{j} - 3\vec{i} - 2\vec{j}$

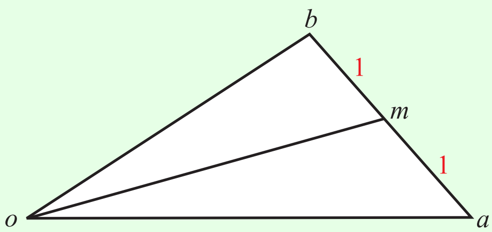
$\therefore \vec{r} = 2\vec{i} - 8\vec{j}$

2 (b) (i)

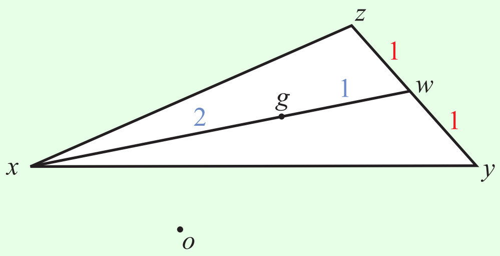
$$\vec{m} = \frac{1\vec{b} + 1\vec{a}}{1+1}$$

$\vec{c} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$ 2

$\therefore \vec{m} = \frac{1}{2}(\vec{a} + \vec{b})$



2 (b) (ii)



$$\vec{w} = \frac{1\vec{y} + 1\vec{z}}{1+1}$$

$$\therefore \vec{w} = \tfrac{1}{2}(\vec{y} + \vec{z})$$

$$\vec{g} = \frac{2\vec{w} + 1\vec{x}}{2+1} = \tfrac{1}{3}(2\vec{w} + \vec{x})$$

$$\Rightarrow \vec{g} = \tfrac{1}{3}(2[\tfrac{1}{2}(\vec{y} + \vec{z})] + \vec{x})$$

$$\therefore \vec{g} = \tfrac{1}{3}(\vec{x} + \vec{y} + \vec{z})$$

$\vec{c} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$ **2**

2 (c) (i)

Let $\vec{r} = x\vec{i} + y\vec{j} \Rightarrow \vec{r}^{\perp} = -y\vec{i} + x\vec{j}$

Let $\vec{s} = p\vec{i} + q\vec{j} \Rightarrow \vec{s}^{\perp} = -q\vec{i} + p\vec{j}$

$\vec{r} = +x\vec{i} + y\vec{j} \Rightarrow \vec{r}^{\perp} = -y\vec{i} + x\vec{j}$ **7**

When you dot two vectors, just multiply the $\vec{i}$ coefficients and multiply the $\vec{j}$ coefficients.

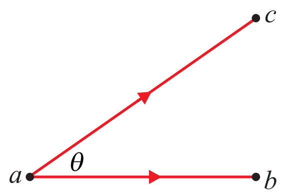
$$\vec{r}.\vec{s}^{\perp} = (x\vec{i} + y\vec{j}).(-q\vec{i} + p\vec{j}) = -xq + yp$$

$$-\vec{r}^{\perp}.\vec{s} = -(-y\vec{i} + x\vec{j}).(p\vec{i} + q\vec{j}) = -(-yp + xq) = -xq + yp$$

$$\therefore \vec{r}.\vec{s}^{\perp} = -\vec{r}^{\perp}.\vec{s}$$

2 (c) (ii)

$\vec{ab}.\vec{ac} = |\vec{ab}||\vec{ac}|\cos\theta$ **8**



Remember it as:

ab dot *ac* = Length [*ab*] × Length [*ac*] × cos of angle between *ab* and *ac*

$$\vec{ab}.\vec{ad} = |\vec{ab}||\vec{ad}|\cos|\angle bad| \Rightarrow \cos|\angle bad| = \frac{\vec{ab}.\vec{ad}}{|\vec{ab}||\vec{ad}|}$$

$$\vec{ac}.\vec{ad} = |\vec{ac}||\vec{ad}|\cos|\angle cad| \Rightarrow \cos|\angle cad| = \frac{\vec{ac}.\vec{ad}}{|\vec{ac}||\vec{ad}|}$$

Therefore, you need to prove that $\dfrac{\vec{ab}.\vec{ad}}{|\vec{ab}||\vec{ad}|} = \dfrac{\vec{ac}.\vec{ad}}{|\vec{ac}||\vec{ad}|}$.

$$\frac{\overrightarrow{ab}.\overrightarrow{ad}}{|\overrightarrow{ab}||\overrightarrow{ad}|}=\frac{1}{|\overrightarrow{ab}||\overrightarrow{ad}|}\overrightarrow{ab}.t\left(\frac{\overrightarrow{ab}^{\perp}}{|\overrightarrow{ab}|}-\frac{\overrightarrow{ac}^{\perp}}{|\overrightarrow{ac}|}\right)$$

$$\Rightarrow\frac{\overrightarrow{ab}.\overrightarrow{ad}}{|\overrightarrow{ab}||\overrightarrow{ad}|}=\frac{t}{|\overrightarrow{ab}||\overrightarrow{ad}|}\left(\frac{\overrightarrow{ab}.\overrightarrow{ab}^{\perp}}{|\overrightarrow{ab}|}-\frac{\overrightarrow{ab}.\overrightarrow{ac}^{\perp}}{|\overrightarrow{ac}|}\right)$$ [If two vectors are perpendicular their dot product is zero.]

$$\Rightarrow\frac{\overrightarrow{ab}.\overrightarrow{ad}}{|\overrightarrow{ab}||\overrightarrow{ad}|}=\frac{t}{|\overrightarrow{ab}||\overrightarrow{ad}|}\left(\frac{0}{|\overrightarrow{ab}|}-\frac{\overrightarrow{ab}.\overrightarrow{ac}^{\perp}}{|\overrightarrow{ac}|}\right)$$

$$\therefore\frac{\overrightarrow{ab}.\overrightarrow{ad}}{|\overrightarrow{ab}||\overrightarrow{ad}|}=-\frac{t\,\overrightarrow{ab}.\overrightarrow{ac}^{\perp}}{|\overrightarrow{ab}||\overrightarrow{ad}||\overrightarrow{ac}|}$$

$$\frac{\overrightarrow{ac}.\overrightarrow{ad}}{|\overrightarrow{ac}||\overrightarrow{ad}|}=\frac{1}{|\overrightarrow{ac}||\overrightarrow{ad}|}\overrightarrow{ac}.t\left(\frac{\overrightarrow{ab}^{\perp}}{|\overrightarrow{ab}|}-\frac{\overrightarrow{ac}^{\perp}}{|\overrightarrow{ac}|}\right)$$

$$\Rightarrow\frac{\overrightarrow{ac}.\overrightarrow{ad}}{|\overrightarrow{ac}||\overrightarrow{ad}|}=\frac{t}{|\overrightarrow{ac}||\overrightarrow{ad}|}\left(\frac{\overrightarrow{ac}.\overrightarrow{ab}^{\perp}}{|\overrightarrow{ab}|}-\frac{\overrightarrow{ac}.\overrightarrow{ac}^{\perp}}{|\overrightarrow{ac}|}\right)$$ [If two vectors are perpendicular their dot product is zero.]

$$\Rightarrow\frac{\overrightarrow{ac}.\overrightarrow{ad}}{|\overrightarrow{ac}||\overrightarrow{ad}|}=\frac{t}{|\overrightarrow{ac}||\overrightarrow{ad}|}\left(\frac{\overrightarrow{ac}.\overrightarrow{ab}^{\perp}}{|\overrightarrow{ab}|}-\frac{0}{|\overrightarrow{ac}|}\right)$$

$$\therefore\frac{\overrightarrow{ac}.\overrightarrow{ad}}{|\overrightarrow{ac}||\overrightarrow{ad}|}=\frac{t\,\overrightarrow{ac}.\overrightarrow{ab}^{\perp}}{|\overrightarrow{ac}||\overrightarrow{ad}||\overrightarrow{ab}|}$$ [You proved in part **(i)** that $\overrightarrow{ac}.\overrightarrow{ab}^{\perp}=-\overrightarrow{ab}.\overrightarrow{ac}^{\perp}$.]

$$\therefore\frac{\overrightarrow{ac}.\overrightarrow{ad}}{|\overrightarrow{ac}||\overrightarrow{ad}|}=-\frac{t\,\overrightarrow{ac}^{\perp}.\overrightarrow{ab}}{|\overrightarrow{ac}||\overrightarrow{ad}||\overrightarrow{ab}|}$$

$$\therefore\frac{\overrightarrow{ac}.\overrightarrow{ad}}{|\overrightarrow{ac}||\overrightarrow{ad}|}=\frac{\overrightarrow{ab}.\overrightarrow{ad}}{|\overrightarrow{ab}||\overrightarrow{ad}|}$$

$$\therefore|\angle bad|=|\angle cad|$$