VECTORS (Q 2, PAPER 2)

1998

2 (a) $abcd$ is a parallelogram where $\vec{a} = 2\vec{i} - 7\vec{j}, \vec{b} = -6\vec{i} - 11\vec{j}$ and $\vec{c} = -8\vec{i} + 4\vec{j}$.

Express $\vec{d}$ in terms of $\vec{i}$ and $\vec{j}$.

(b) $\vec{p} = 9\vec{i} - 5\vec{j}, \vec{q} = 5\vec{i} + 3\vec{j}$ and $\vec{s} = -5\vec{i} - \frac{9}{2}\vec{j}$.

Let $\vec{m} = \frac{1}{2}(\vec{p} + \vec{q})$ and $\vec{n} = \frac{2}{5}(\overrightarrow{sq})$.

(i) Express $\vec{m}$ and $\vec{n}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the measure of the angle between $\vec{m}$ and $\vec{n}$.

(c) $\vec{x} = -3\vec{i} + 4\vec{j}$ and $\vec{y} = 5\vec{i} + 12\vec{j}$.

(i) Find $|\vec{x}|$ and $|\vec{y}|$.

(ii) If $\vec{r} = (1-t)\vec{x} + t\vec{y}$, where $t = \dfrac{|\vec{x}|}{|\vec{x}| + |\vec{y}|}$,

express $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

(iii) If $k\left(\dfrac{\vec{x}}{|\vec{x}|} + \dfrac{\vec{y}}{|\vec{y}|}\right) = 18\vec{r}$, find the value of k, $k \in \mathbf{N}$.

SOLUTION

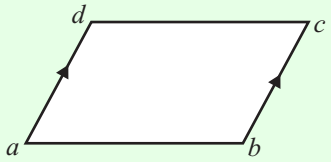
2 (a)

$\overrightarrow{ad} = \overrightarrow{bc} \Rightarrow \vec{d} - \vec{a} = \vec{c} - \vec{b}$

$\Rightarrow \vec{d} = \vec{a} - \vec{b} + \vec{c}$

$\Rightarrow \vec{d} = 2\vec{i} - 7\vec{j} + 6\vec{i} + 11\vec{j} - 8\vec{i} + 4\vec{j}$

$\therefore \vec{d} = 0\vec{i} + 8j$



$\overrightarrow{ab} = \vec{b} - \vec{a}$ 1

2 (b) (i)

$\vec{m} = \frac{1}{2}(\vec{p} + \vec{q})$

$\Rightarrow \vec{m} = \frac{1}{2}(9\vec{i} - 5\vec{j} + 5\vec{i} + 3\vec{j})$

$\Rightarrow \vec{m} = \frac{1}{2}(14\vec{i} - 2\vec{j})$

$\therefore \vec{m} = 7\vec{i} - \vec{j}$

$\overrightarrow{ab} = \vec{b} - \vec{a}$ 1

$\vec{n} = \frac{2}{5}(\overrightarrow{sq}) \Rightarrow \vec{n} = \frac{2}{5}(\vec{q} - \vec{s})$

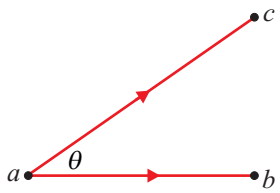
$\Rightarrow \vec{n} = \frac{2}{5}(5\vec{i} + 3\vec{j} + 5\vec{i} + \frac{9}{2}\vec{j})$

$\Rightarrow \vec{n} = \frac{2}{5}(10\vec{i} + \frac{15}{2}\vec{j})$

$\therefore \vec{n} = 4\vec{i} + 3\vec{j}$

2 (b) (ii)

$\overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos\theta$ 8



Remember it as:

ab dot *ac* = Length [*ab*] × Length [*ac*] × cos of angle between *ab* and *ac*

$\vec{m}.\vec{n} = |\vec{m}||\vec{n}|\cos\theta$

$\vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2}$ 5

$$\Rightarrow \cos\theta = \frac{\vec{m}.\vec{n}}{|\vec{m}||\vec{n}|} = \frac{(7\vec{i} - \vec{j}).(4\vec{i} + 3\vec{j})}{|7\vec{i} - \vec{j}||4\vec{i} + 3\vec{j}|}$$

$$\Rightarrow \cos\theta = \frac{28 - 3}{\sqrt{49+1}\sqrt{16+9}} = \frac{25}{\sqrt{50}\sqrt{25}}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = 45^{\circ}$$

2 (c) (i)

$$|\vec{x}| = \sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5$$

$$|\vec{y}| = \sqrt{(5)^2 + (12)^2} = \sqrt{169} = 13$$

$\vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2}$ 5

2 (c) (ii)

$$t = \frac{|\vec{x}|}{|\vec{x}| + |\vec{y}|} = \frac{5}{5+13} = \frac{5}{18}$$

$$\vec{r} = (1-t)\vec{x} + t\vec{y} = \tfrac{13}{18}\vec{x} + \tfrac{5}{18}\vec{y}$$

$$\Rightarrow \vec{r} = \tfrac{13}{18}(-3\vec{i} + 4\vec{j}) + \tfrac{5}{18}(5\vec{i} + 12\vec{j})$$

$$\Rightarrow \vec{r} = -\tfrac{39}{18}\vec{i} + \tfrac{52}{18}\vec{j} + \tfrac{25}{18}\vec{i} + \tfrac{60}{18}\vec{j}$$

$$\Rightarrow \vec{r} = -\tfrac{14}{18}\vec{i} + \tfrac{112}{18}\vec{j}$$

$$\therefore \vec{r} = -\tfrac{7}{9}\vec{i} + \tfrac{56}{9}\vec{j}$$

2 (c) (iii)

$$k\left(\frac{\vec{x}}{|\vec{x}|} + \frac{\vec{y}}{|\vec{y}|}\right) = 18\vec{r} \Rightarrow k\left(\frac{-3\vec{i} + 4\vec{j}}{|-3\vec{i} + 4\vec{j}|} + \frac{5\vec{i} + 12\vec{j}}{|5\vec{i} + 12\vec{j}|}\right) = 18[-\tfrac{7}{9}\vec{i} + \tfrac{56}{9}\vec{j}]$$

$$\Rightarrow k\left(\frac{-3\vec{i} + 4\vec{j}}{5} + \frac{5\vec{i} + 12\vec{j}}{13}\right) = -14\vec{i} + 112\vec{j}$$

$$\Rightarrow k\left(\frac{-39\vec{i} + 52\vec{j} + 25\vec{i} + 60\vec{j}}{65}\right) = -14\vec{i} + 112\vec{j}$$

$$\Rightarrow k\left(\frac{14\vec{i} + 112\vec{j}}{65}\right) = -14\vec{i} + 112\vec{j}$$

$$\therefore k = 65$$