VECTORS (Q 2, PAPER 2)

1997

2 (a) $oabc$ is a parallelogram where o is the origin.

If $\vec{a} = 6\vec{i} - 2\vec{j}$ and $\vec{b} = 2\vec{i} - 5\vec{j}$, express $\vec{c}$ in terms of $\vec{i}$ and $\vec{j}$.

(b) $\vec{p} = 2\vec{i} + 3\vec{j}$ and $\vec{p}^{\perp}$ is its related vector $-3\vec{i} + 2\vec{j}$.

Let $\vec{q} = \vec{p}^{\perp} - \vec{p}$ and $\vec{r} = \vec{q} + \vec{q}^{\perp}$.

(i) Express $\vec{q}$ and $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find the measure of the angle between $\vec{q}$ and $\vec{r}$.

(c) $\vec{o}, \vec{x}$ and $\vec{y}$ are non-collinear vectors where o is the origin.

(i) Show $\vec{x}.\vec{x} = |\vec{x}|^2$ and $\vec{x}.\vec{y} = \vec{y}.\vec{x}$.

(ii) If $|\vec{x} + \vec{y}| = |\vec{x} - \vec{y}|$, prove that $\vec{x} \perp \vec{y}$.

SOLUTION

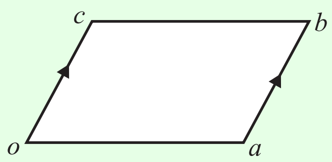
2 (a)

$\overrightarrow{oc} = \overrightarrow{ab} \Rightarrow \vec{c} = \vec{b} - \vec{a}$

$\Rightarrow \vec{c} = 2\vec{i} - 5\vec{j} - 6\vec{i} + 2\vec{j}$

$\therefore \vec{c} = -4\vec{i} - 3\vec{j}$

$\overrightarrow{ab} = \vec{b} - \vec{a}$ 1



2 (b) (i)

$\vec{p} = 2\vec{i} + 3\vec{j}, \ \vec{p}^{\perp} = -3\vec{i} + 2\vec{j}$

$\vec{q} = \vec{p}^{\perp} - \vec{p} = -3\vec{i} + 2\vec{j} - 2\vec{i} - 3\vec{j}$

$\therefore \vec{q} = -5\vec{i} - \vec{j}$

$\therefore \vec{q}^{\perp} = \vec{i} - 5\vec{j}$

$\vec{r} = +x\vec{i} + y\vec{j} \Rightarrow \vec{r}^{\perp} = -y\vec{i} + x\vec{j}$ 7

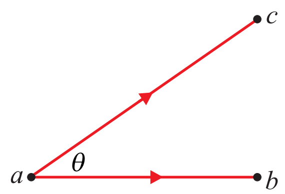
$\vec{r} = \vec{q} + \vec{q}^{\perp}$

$\Rightarrow \vec{r} = -5\vec{i} - \vec{j} + \vec{i} - 5\vec{j}$

$\therefore \vec{r} = -4\vec{i} - 6\vec{j}$

2 (b) (ii)

$\overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos\theta$ 8



Remember it as:

ab dot *ac* = Length [*ab*] × Length [*ac*] × cos of angle between *ab* and *ac*

$\vec{q}.\vec{r} = |\vec{q}||\vec{r}|\cos\theta$

$\Rightarrow \cos\theta = \dfrac{\vec{q}.\vec{r}}{|\vec{q}||\vec{r}|}$

$\Rightarrow \cos\theta = \dfrac{(-5\vec{i} - \vec{j}).(-4\vec{i} - 6\vec{j})}{|-5\vec{i} - \vec{j}||-4\vec{i} - 6\vec{j}|}$

When you dot two vectors, just multiply the $\vec{i}$ coefficients and multiply the $\vec{j}$ coefficients.

$\Rightarrow \cos\theta = \dfrac{20+6}{\sqrt{25+1}\sqrt{16+36}} = \dfrac{26}{\sqrt{26}\sqrt{52}}$

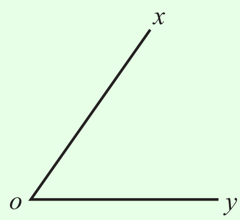
$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}}$

$\therefore \theta = 45^\circ$

2 (c) (i)

$\vec{x}.\vec{x} = |\vec{x}||\vec{x}|\cos 0^\circ = |\vec{x}|^2$

$\overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos\theta$ 8



$\vec{x}.\vec{y} = |\vec{x}||\vec{y}|\cos\theta$

$\vec{y}.\vec{x} = |\vec{y}||\vec{x}|\cos\theta = |\vec{x}||\vec{y}|\cos\theta$

$\therefore \vec{x}.\vec{y} = \vec{y}.\vec{x}$

2 (c) (ii)

$|\vec{x}+\vec{y}| = |\vec{x}-\vec{y}|$

$\Rightarrow |\vec{x}+\vec{y}|^2 = |\vec{x}-\vec{y}|^2$

$\Rightarrow (\vec{x}+\vec{y}).(\vec{x}+\vec{y}) = (\vec{x}-\vec{y}).(\vec{x}-\vec{y})$ [Using the result in part **(i)**.]

$\Rightarrow \vec{x}.\vec{x} + \vec{x}.\vec{y} + \vec{y}.\vec{x} + \vec{y}.\vec{y} = \vec{x}.\vec{x} - \vec{x}.\vec{y} - \vec{y}.\vec{x} + \vec{y}.\vec{y}$

$\Rightarrow 2\vec{x}.\vec{y} = -2\vec{x}.\vec{y}$ [Using the result in part **(i)**.]

$\Rightarrow 4\vec{x}.\vec{y} = 0$

$\Rightarrow \vec{x}.\vec{y} = 0$ $\quad \overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos\theta$ 8

$\Rightarrow |\vec{x}||\vec{y}|\cos 90^\circ = 0$

$\therefore \vec{x} \perp \vec{y}$ [If two vectors are perpendicular their dot product is zero.]