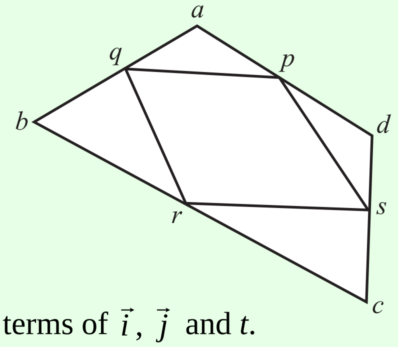
VECTORS (Q 2, PAPER 2)

1996

2 (a) $\vec{r} = 7\vec{i} - 4\vec{j}$ and $\vec{r}^{\perp}$ is its related vector $4\vec{i} + 7\vec{j}$.

$m\vec{r} + n\vec{r}^{\perp} = 5\vec{i} - 40\vec{j}$. Find the value of m and the value of n where m and $n \in \mathbf{R}$.

(b) p, q, r, s are the mid-points of the sides of a quadrilateral $abcd$.
Prove by vector methods that $pqrs$ is a parallogram.



(c) o is the origin, $\vec{a} = 2\vec{i} + 2\vec{j}$, $\vec{b} = 4\vec{i} + 4\vec{j}$.

If $\vec{r} = \frac{1}{2}(\vec{a} + \vec{b}) + t(\vec{b} - \vec{a})^{\perp}$, $t \in \mathbf{R}$, express $\vec{r}$ in terms of $\vec{i}$, $\vec{j}$ and t.

Show that r lies on the perpendicular bisector of $[ab]$ for all $t \in \mathbf{R}$,

i.e. show that $|\vec{ra}| = |\vec{rb}|$.

SOLUTION

2 (a)

$\vec{r} = 7\vec{i} - 4\vec{j}$, $\vec{r}^{\perp} = 4\vec{i} + 7\vec{j}$

$m\vec{r} + n\vec{r}^{\perp} = 5\vec{i} - 40\vec{j}$

$\Rightarrow m(7\vec{i} - 4\vec{j}) + n(4\vec{i} + 7\vec{j}) = 5\vec{i} - 40\vec{j}$

$\Rightarrow 7m\vec{i} - 4m\vec{j} + 4n\vec{i} + 7n\vec{j} = 5\vec{i} - 40\vec{j}$

$\Rightarrow (7m + 4n)\vec{i} + (-4m + 7n)\vec{j} = 5\vec{i} - 40\vec{j}$

$\therefore 7m + 4n = 5$....(**1**)

$\therefore -4m + 7n = -40$....(**2**)

Solve Eqns. (**1**) and (**2**) simultaneously.

$7m + 4n = 5$........(**1**)($\times 4$)	$28m + 16n = 20$
$-4m + 7n = -40$....(**2**)($\times 7$)	$-28m + 49n = -280$
	$65n = -260 \Rightarrow n = -4$

Substitute this value of n into Eqn. (**1**):

$7m + 4(-4) = 5 \Rightarrow 7m - 16 = 5$

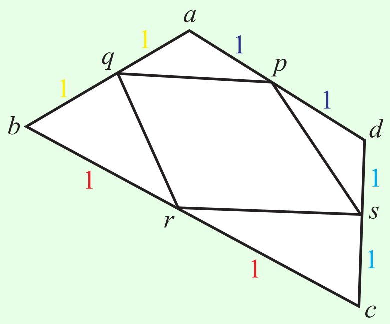
$\Rightarrow 7m = 21$

$\therefore m = 3$

2 (b)

$\vec{p} = \dfrac{1\vec{d} + 1\vec{a}}{1+1} = \tfrac{1}{2}(\vec{a} + \vec{d})$

$\vec{c} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$ 2



$\vec{s} = \dfrac{1\vec{c} + 1\vec{d}}{1+1} = \tfrac{1}{2}(\vec{c} + \vec{d})$

$\vec{r} = \dfrac{1\vec{b} + 1\vec{c}}{1+1} = \tfrac{1}{2}(\vec{b} + \vec{c})$

$\vec{q} = \dfrac{1\vec{b} + 1\vec{a}}{1+1} = \tfrac{1}{2}(\vec{a} + \vec{b})$

$\overrightarrow{qp} = \vec{p} - \vec{q} = \tfrac{1}{2}(\vec{a} + \vec{d}) - \tfrac{1}{2}(\vec{a} + \vec{b}) = \tfrac{1}{2}(\vec{d} - \vec{b})$

$\overrightarrow{rs} = \vec{s} - \vec{r} = \tfrac{1}{2}(\vec{c} + \vec{d}) - \tfrac{1}{2}(\vec{b} + \vec{c}) = \tfrac{1}{2}(\vec{d} - \vec{b})$

$\therefore \overrightarrow{qp} = \overrightarrow{rs}$

Therefore, [*qp*] is parallel to and equal in length to [*rs*]. Therefore, *pqrs* is a parallelogram.

2 (c)

$\vec{r} = \tfrac{1}{2}(\vec{a} + \vec{b}) + t(\vec{b} - \vec{a})^{\perp}$

$\Rightarrow \vec{r} = \tfrac{1}{2}(2\vec{i} + 2\vec{j} + 4\vec{i} + 4\vec{j}) + t(4\vec{i} + 4\vec{j} - 2\vec{i} - 2\vec{j})^{\perp}$

$\Rightarrow \vec{r} = \tfrac{1}{2}(6\vec{i} + 6\vec{j}) + t(2\vec{i} + 2\vec{j})^{\perp}$

$\Rightarrow \vec{r} = (3\vec{i} + 3\vec{j}) + t(-2\vec{i} + 2\vec{j})$ $\quad \vec{r} = +x\vec{i} + y\vec{j} \Rightarrow \vec{r}^{\perp} = -y\vec{i} + x\vec{j}$ 7

$\therefore \vec{r} = (3-2t)\vec{i} + (3+2t)\vec{j}$

$|\overrightarrow{ra}| = |\vec{a} - \vec{r}| = |2\vec{i} + 2\vec{j} - (3-2t)\vec{i} - (3+2t)\vec{j}|$

$\Rightarrow |\overrightarrow{ra}| = |2\vec{i} + 2\vec{j} - 3\vec{i} + 2t\vec{i} - 3\vec{j} - 2t\vec{j}|$

$\Rightarrow |\overrightarrow{ra}| = |(2t-1)\vec{i} + (-2t-1)\vec{j}|$ $\quad \vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2}$ 5

$\Rightarrow |\overrightarrow{ra}| = \sqrt{(2t-1)^2 + (-2t-1)^2}$

$\Rightarrow |\overrightarrow{ra}| = \sqrt{4t^2 - 4t + 1 + 4t^2 + 4t + 1}$

$\Rightarrow |\overrightarrow{ra}| = \sqrt{8t^2 + 2}$

$|\overrightarrow{rb}| = |\vec{b} - \vec{r}| = |4\vec{i} + 4\vec{j} - (3-2t)\vec{i} - (3+2t)\vec{j}|$

$\Rightarrow |\overrightarrow{rb}| = |4\vec{i} + 4\vec{j} - 3\vec{i} + 2t\vec{i} - 3\vec{j} - 2t\vec{j}|$

$\Rightarrow |\overrightarrow{rb}| = |(2t+1)\vec{i} + (-2t+1)\vec{j}|$

$\Rightarrow |\overrightarrow{rb}| = \sqrt{(2t+1)^2 + (-2t+1)^2}$

$\Rightarrow |\overrightarrow{rb}| = \sqrt{4t^2 + 4t + 1 + 4t^2 - 4t + 1}$

$\Rightarrow |\overrightarrow{rb}| = \sqrt{8t^2 + 2}$

$\therefore |\overrightarrow{ra}| = |\overrightarrow{rb}|$