# VECTORS (Q 2, PAPER 2)

## 2006

2 (a) $\vec{x} = -3\vec{i} + \vec{j}$. Express $(\vec{x}^{\perp})^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

2 (b) $\vec{p} = -5\vec{i} + 2\vec{j}$, $\vec{q} = \vec{i} - 6\vec{j}$ and $\vec{r} = -\vec{i} + 5\vec{j}$.

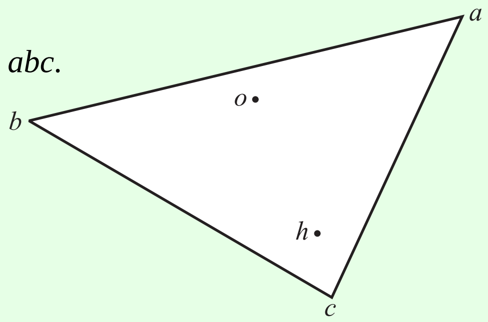
(i) Express $\overrightarrow{pq}$ and $\overrightarrow{pr}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Given that $10\vec{s} = |\overrightarrow{pr}|\,\overrightarrow{pq} + |\overrightarrow{pq}|\,\overrightarrow{pr}$, express $\vec{s}$ in terms of $\vec{i}$ and $\vec{j}$.

(iii) Find the measure of the angle between $\vec{s}$ and $\overrightarrow{pr}$.

2 (c) The origin $o$ is the circumcentre of the triangle $abc$.

If $\vec{h} = \vec{a} + \vec{b} + \vec{c}$, show that $\overrightarrow{ah} \perp \overrightarrow{bc}$.




**SOLUTION**

**2 (a)**

$\vec{r} = +x\vec{i} + y\vec{j} \Rightarrow \vec{r}^{\perp} = -y\vec{i} + x\vec{j}$ ....... 7

$\vec{x} = -3\vec{i} + \vec{j} \Rightarrow \vec{x}^{\perp} = -\vec{i} - 3\vec{j}$

$\Rightarrow (\vec{x}^{\perp})^{\perp} = 3\vec{i} - \vec{j}$

**2 (b) (i)**

$\overrightarrow{ab} = \vec{b} - \vec{a}$ ....... 1

$\overrightarrow{pq} = \vec{q} - \vec{p} = (\vec{i} - 6\vec{j}) - (-5\vec{i} + 2\vec{j}) = 6\vec{i} - 8\vec{j}$

$\overrightarrow{pr} = \vec{r} - \vec{p} = (-\vec{i} + 5\vec{j}) - (-5\vec{i} + 2\vec{j}) = 4\vec{i} + 3\vec{j}$

**2(b) (ii)**

$\vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2}$ ....... 5

$|\overrightarrow{pr}| = |4\vec{i} + 3\vec{j}| = \sqrt{16 + 9} = \sqrt{25} = 5$

$|\overrightarrow{pq}| = |6\vec{i} - 8\vec{j}| = \sqrt{36 + 64} = \sqrt{100} = 10$

$\therefore 10\vec{s} = |\overrightarrow{pr}|\,\overrightarrow{pq} + |\overrightarrow{pq}|\,\overrightarrow{pr} \Rightarrow 10\vec{s} = 5(6\vec{i} - 8\vec{j}) + 10(4\vec{i} + 3\vec{j})$

$\Rightarrow 10\vec{s} = 30\vec{i} - 40\vec{j} + 40\vec{i} + 30\vec{j} \Rightarrow 10\vec{s} = 70\vec{i} - 10\vec{j}$

$\Rightarrow \vec{s} = 7\vec{i} - \vec{j}$

**2 (b) (iii)**

$\vec{s}.\overrightarrow{pr} = |\vec{s}||\overrightarrow{pr}|\cos\theta$

$\Rightarrow (7\vec{i} - \vec{j}).(4\vec{i} + 3\vec{j}) = |7\vec{i} - \vec{j}||4\vec{i} + 3\vec{j}|\cos\theta$

$\Rightarrow 25 = \sqrt{50}\sqrt{25}\cos\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$

$\overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos\theta$ ....... **8**

$\vec{i}.\vec{j} = \vec{j}.\vec{i} = 0$ ....... **9**

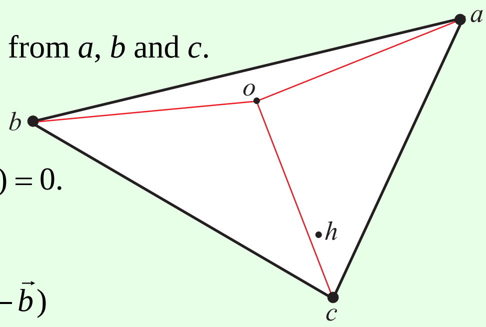
$\vec{i}.\vec{i} = \vec{j}.\vec{j} = |\vec{i}|^2 = |\vec{j}|^2 = 1$ ...... **10**

**Trick**: Just multiply the $\vec{i}$ coefficients and multiply the $\vec{j}$ coefficients.

**2 (c)**

**Dot Product Properties**

1. $\overrightarrow{ab}.\overrightarrow{ac} = \overrightarrow{ac}.\overrightarrow{ab}$ [You can switch the order of dot product]
2. $\theta = 90^\circ \Leftrightarrow \overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos 90^\circ = 0$
   [If two vectors are perpendicular their dot product is zero. If their dot product is zero, the vectors are perpendicular.]
3. $\overrightarrow{ab} = \overrightarrow{ac} \Leftrightarrow \overrightarrow{ab}.\overrightarrow{ab} = |\overrightarrow{ab}||\overrightarrow{ab}|\cos 0^\circ = |\overrightarrow{ab}|^2$
   [If you dot product a vector with itself you get the modulus squared. The modulus squared of a vector is got by dotting it with itself.]




If $o$ is the circumcentre of triangle $abc$, $o$ is equidistant from $a$, $b$ and $c$.

$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}|$

To prove that $\overrightarrow{ah} \perp \overrightarrow{bc}$, you need to show that $(\overrightarrow{ah}).(\overrightarrow{bc}) = 0$.

$(\overrightarrow{ah}).(\overrightarrow{bc}) = (\vec{h} - \vec{a}).(\vec{c} - \vec{b})$

But $\vec{h} = \vec{a} + \vec{b} + \vec{c} \Rightarrow (\vec{a} + \vec{b} + \vec{c} - \vec{a}).(\vec{c} - \vec{b}) = (\vec{b} + \vec{c}).(\vec{c} - \vec{b})$

$= \vec{b}.\vec{c} - \vec{b}.\vec{b} + \vec{c}.\vec{c} - \vec{c}.\vec{b} = |\vec{c}|^2 - |\vec{b}|^2 = 0$