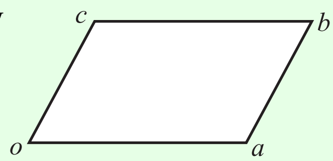
VECTORS (Q 2, PAPER 2)

2005

2 (a) Copy the parallelogram oabc in your answerbook. Show your work, construct the point d such that

$\vec{d} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} - \vec{c}$, where o is the origin.



2 (b) $\vec{p} = 3\vec{i} + 4\vec{j}$. $\vec{q}$ is the unit vector in the direction of $\vec{p}$.

(i) Express $\vec{q}$ and $\vec{q}^{\perp}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Express $11\vec{i} - 2\vec{j}$ in the form $k\vec{q} + l\vec{q}^{\perp}$, where $k, l \in \mathbf{R}$.

2 (c) $\vec{u} = \vec{i} + 5\vec{j}$ and $\vec{v} = 4\vec{i} + 4\vec{j}$.

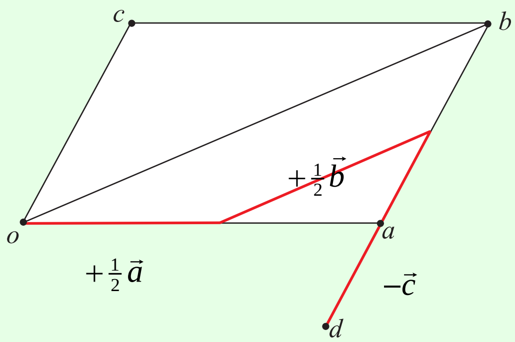
(i) Find $\cos \angle uov$, where o is the origin.

(ii) $\vec{r} = (1-k)\vec{u} + k\vec{v}$, where $k \in \mathbf{R}$ and $k \neq 0$. Find the value of k for which

$|\angle uov| = |\angle vor|$.

SOLUTION

2 (a)



2 (b) (i)

$$\vec{p} = 3\vec{i} + 4\vec{j} \Rightarrow \vec{q} = \frac{\vec{p}}{|\vec{p}|} = \frac{3\vec{i} + 4\vec{j}}{\sqrt{9+16}} = \tfrac{3}{5}\vec{i} + \tfrac{4}{5}\vec{j}$$

Unit Vector: $\frac{\vec{r}}{|\vec{r}|} = \frac{x\vec{i} + y\vec{j}}{\sqrt{x^2 + y^2}}$ **6**

$$\vec{p}^{\perp} = -\tfrac{4}{5}\vec{i} + \tfrac{3}{5}\vec{j}$$

$\vec{r} = +x\vec{i} + y\vec{j} \Rightarrow \vec{r}^{\perp} = -y\vec{i} + x\vec{j}$ **7**

2 (b) (ii)

$$11\vec{i} - 2\vec{j} = k\vec{q} + l\vec{q}^{\perp} \Rightarrow 11\vec{i} - 2\vec{j} = k(\tfrac{3}{5}\vec{i} + \tfrac{4}{5}\vec{j}) + l(-\tfrac{4}{5}\vec{i} + \tfrac{3}{5}\vec{j})$$

Equate the i and j parts: $\therefore 11 = \frac{3}{5}k - \frac{4}{5}l$ and $-2 = \frac{4}{5}k + \frac{3}{5}l$

Solve simultaneously for k and l:

$$11 = \tfrac{3}{5}k - \tfrac{4}{5}l \Rightarrow 55 = 3k - 4l \ldots\mathbf{(1)}\ (\times 3)$$
$$-2 = \tfrac{4}{5}k + \tfrac{3}{5}l \Rightarrow -10 = 4k + 3l \ldots\mathbf{(2)}\ (\times 4)$$

$\rightarrow$

$$165 = 9k - 12l$$
$$-40 = 16k + 12l$$
$$125 = 25k \quad \Rightarrow k = 5$$

Substitute this value of k into equation **1**: $\therefore 55 = 3(5) - 4l \Rightarrow 40 = -4l \Rightarrow l = -10$

Ans: $5\vec{q} - 10\vec{q}^{\perp}$

2 (c) (i)

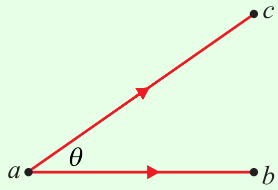
$\vec{ou} = \vec{i} + 5\vec{j}, \ \vec{ov} = 4\vec{i} + 4\vec{j}$

$\vec{ou}.\vec{ov} = |\vec{ou}||\vec{ov}|\cos\angle uov$

$\Rightarrow (\vec{i} + 5\vec{j}).(4\vec{i} + 4\vec{j}) = |\vec{i} + 5\vec{j}||4\vec{i} + 4\vec{j}|\cos\angle uov$

$\Rightarrow 24 = \sqrt{26}\sqrt{32}\cos\angle uov \Rightarrow \cos\angle uov = \dfrac{24}{4\sqrt{26}\sqrt{2}} = \dfrac{3}{\sqrt{13}}$

$\vec{ab}.\vec{ac} = |\vec{ab}||\vec{ac}|\cos\theta$ 8



$\vec{i}.\vec{j} = \vec{j}.\vec{i} = 0$ 9

Trick: Just multiply the $\vec{i}$ coefficients and multiply the $\vec{j}$ coefficients.

$\vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2}$ 5

2 (c) (ii)

$\vec{or} = (1-k)\vec{u} + k\vec{v}$

$\Rightarrow \vec{or} = (1-k)(\vec{i} + 5\vec{j}) + k(4\vec{i} + 4\vec{j}) = (3k+1)\vec{i} + (5-k)\vec{j}$

If $|\angle uov| = |\angle vor| \Rightarrow \cos|\angle uov| = \cos|\angle vor| \Rightarrow \cos|\angle vor| = \frac{3}{\sqrt{13}}$

$\vec{or}.\vec{ov} = |\vec{or}||\vec{ov}|\cos\angle vor \Rightarrow \cos\angle vor = \dfrac{\vec{or}.\vec{ov}}{|\vec{or}||\vec{ov}|}$

$\Rightarrow \dfrac{3}{\sqrt{13}} = \dfrac{[(3k+1)\vec{i} + (5-k)\vec{j}].[4\vec{i} + 4\vec{j}]}{|(3k+1)\vec{i} + (5-k)\vec{j}||4\vec{i} + 4\vec{j}|} \Rightarrow \dfrac{3}{\sqrt{13}} = \dfrac{4(3k+1) + 4(5-k)}{\sqrt{(3k+1)^2 + (5-k)^2}\sqrt{32}}$

$\Rightarrow \dfrac{3}{\sqrt{13}} = \dfrac{12k + 4 + 20 - 4k}{4\sqrt{(3k+1)^2 + (5-k)^2}\sqrt{2}} \Rightarrow \dfrac{3}{\sqrt{13}} = \dfrac{8k+24}{4\sqrt{(3k+1)^2 + (5-k)^2}\sqrt{2}}$

$\Rightarrow \dfrac{3}{\sqrt{13}} = \dfrac{2k+6}{\sqrt{(3k+1)^2 + (5-k)^2}\sqrt{2}} \Rightarrow 3\sqrt{(3k+1)^2 + (5-k)^2}\sqrt{2} = (2k+6)\sqrt{13}$

$\Rightarrow 18[(3k+1)^2 + (5-k)^2] = 13(2k+6)^2$

$\Rightarrow 18[10k^2 - 4k + 26] = 13(4k^2 + 24k + 36)$

$\Rightarrow 180k^2 - 72k + 468 = 52k^2 + 312k + 468$

$\Rightarrow 128k^2 - 384k = 0 \Rightarrow k^2 - 3k = 0 \Rightarrow k(k-3) = 0$

$\Rightarrow k = 3$. (You were told that $k \neq 0$.)