VECTORS (Q 2, PAPER 2)

2004

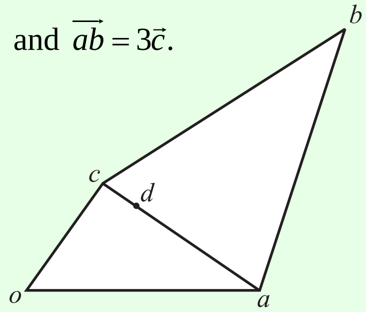
2 (a) $\vec{r} = 12\vec{i} - 35\vec{j}$. Find the unit vector in the direction of $\vec{r}$.

2 (b) *oabc* is a quadrilateral, where *o* is the origin. $\overrightarrow{ad} = 3\overrightarrow{dc}$ and $\overrightarrow{ab} = 3\vec{c}$.

(i) Express $\vec{d}$ in terms of $\vec{a}$ and $\vec{c}$.

(ii) Express $\overrightarrow{db}$ in terms of $\vec{a}$ and $\vec{c}$.

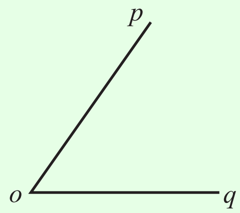
(iii) Show that *o*, *d* and *b* are collinear.



2 (c) *p* and *q* are points and *o* is the origin. *p*, *q* and *o* are not collinear and $|\vec{p}| = |\vec{q}|$.

(i) Prove that $\overrightarrow{pq}$ is perpendicular to $(\vec{p} + \vec{q})$.

(ii) Prove that $\overrightarrow{po}.\overrightarrow{pq} = \frac{1}{2}|\overrightarrow{pq}|^2$.



SOLUTION

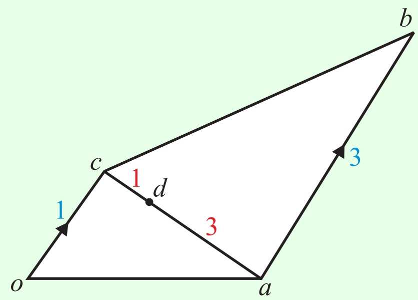
2 (a)

Unit Vector: $\dfrac{\vec{r}}{|\vec{r}|} = \dfrac{x\vec{i} + y\vec{j}}{\sqrt{x^2 + y^2}}$ 6

$\vec{r} = 12\vec{i} - 35\vec{j}$

Unit vector $= \dfrac{\vec{r}}{|\vec{r}|} = \dfrac{12\vec{i} - 35\vec{j}}{\sqrt{12^2 + 35^2}} = \dfrac{12\vec{i} - 35\vec{j}}{37} = \frac{12}{37}\vec{i} - \frac{35}{37}\vec{j}$

2 (b) From the information provided, *d* divides [*ac*] in the ratio 3:1 and [*ab*] is three times longer than [*oc*] and parallel to it.



2 (b) (i)

$\vec{c} = \dfrac{m\vec{b} + n\vec{a}}{m + n}$ 2

$\vec{d} = \dfrac{1\vec{a} + 3\vec{c}}{4} = \frac{1}{4}\vec{a} + \frac{3}{4}\vec{c}$

2 (b) (ii)

$\overrightarrow{ab} = \vec{b} - \vec{a}$ 1

$\overrightarrow{ab} = 3\vec{c} \Rightarrow \vec{b} - \vec{a} = 3\vec{c} \Rightarrow \vec{b} = \vec{a} + 3\vec{c}$

$\overrightarrow{db} = \vec{b} - \vec{d} = \vec{a} + 3\vec{c} - \frac{1}{4}\vec{a} - \frac{3}{4}\vec{c} = \frac{3}{4}\vec{a} + \frac{9}{4}\vec{c}$

2 (b) (iii)

Three points are collinear if a vector formed from any two is a scalar multiplied by a vector formed from any other 2 points.

a, c, b collinear $\Rightarrow \overrightarrow{ac} = k\overrightarrow{ab}$

$\overrightarrow{db} = 3(\frac{1}{4}\vec{a} + \frac{3}{4}\vec{c}) = 3\overrightarrow{od} \Rightarrow o,\ d,\ b$ are collinear.

2 (c) (i)

Dot Product Properties

1. $\overrightarrow{ab}.\overrightarrow{ac} = \overrightarrow{ac}.\overrightarrow{ab}$ [You can switch the order of dot product]
2. $\theta = 90^\circ \Leftrightarrow \overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos 90^\circ = 0$
   [If two vectors are perpendicular their dot product is zero. If their dot product is zero, the vectors are perpendicular.]
3. $\overrightarrow{ab} = \overrightarrow{ac} \Leftrightarrow \overrightarrow{ab}.\overrightarrow{ab} = |\overrightarrow{ab}||\overrightarrow{ab}|\cos 0^\circ = |\overrightarrow{ab}|^2$
   [If you dot product a vector with itself you get the modulus squared. The modulus squared of a vector is got by dotting it with itself.]

$\overrightarrow{pq}.(\vec{p}+\vec{q}) = (\vec{q}-\vec{p}).(\vec{p}+\vec{q}) = \vec{q}.\vec{p} + \vec{q}.\vec{q} - \vec{p}.\vec{p} - \vec{p}.\vec{q}$

$= |\vec{q}|^2 - |\vec{p}|^2 = 0$ as you are told that $|\vec{p}| = |\vec{q}|$.

Therefore, $\overrightarrow{pq}$ is perpendicular to $(\vec{p}+\vec{q})$ as their dot product is zero.

2 (c) (ii)

$\overrightarrow{po}.\overrightarrow{pq} = -\vec{p}.(\vec{q}-\vec{p}) = \vec{p}.\vec{p} - \vec{p}.\vec{q} = |\vec{p}|^2 - \vec{p}.\vec{q}$ (LHS)

$\frac{1}{2}|\overrightarrow{pq}|^2 = \frac{1}{2}\overrightarrow{pq}.\overrightarrow{pq} = \frac{1}{2}(\vec{q}-\vec{p}).(\vec{q}-\vec{p}) = \frac{1}{2}(\vec{q}.\vec{q} - 2\vec{p}.\vec{q} + \vec{p}.\vec{p})$

$= \frac{1}{2}(|\vec{q}|^2 - 2\vec{p}.\vec{q} + |\vec{p}|^2) = \frac{1}{2}(-2\vec{p}.\vec{q} + 2|\vec{p}|^2) = -\vec{p}.\vec{q} + |\vec{p}|^2$ (RHS)