VECTORS (Q 2, PAPER 2)

2003

2 (a) *oabc* is a parallelogram where *o* is the origin, $\vec{a} = 3\vec{i} - \vec{j}$ and $\vec{b} = 4\vec{i} + 3\vec{j}$. Express $\vec{c}$ in terms of $\vec{i}$ and $\vec{j}$.

2 (b) $\vec{p} = 2\vec{i} + \vec{j}$, $\vec{q} = 3\vec{i} + k\vec{j}$, $\vec{r} = 3\vec{i} + t\vec{j}$ where $k, t \in \mathbf{R}$ and *o* is the origin.

(i) Given that $\vec{p} \perp \vec{q}$, calculate the value of *k*.

(ii) Given that $|\angle por| = 45^\circ$, calculate the two possible values of *t*.

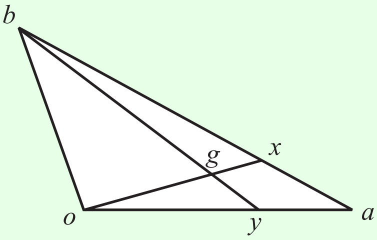
2 (c) *oab* is a triangle where *o* is the origin.

(i) *x* is a point on [*ab*] such that $|ax| : |xb| = 1:3$.

Express $\vec{x}$ in terms of $\vec{a}$ and $\vec{b}$.

(ii) *y* is a point on [*oa*] such that $|oy| : |ya| = 2:1$.

Express $\overrightarrow{by}$ in terms of $\vec{a}$ and $\vec{b}$.

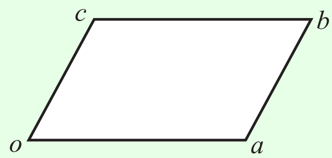


(iii) [*ox*] and [*by*] intersect at *g*. Given that $\vec{g} = m\vec{x}$ and $\overrightarrow{bg} = n\overrightarrow{by}$ where $m, n \in \mathbf{R}$, find the value of *m* and the value of *n*.

SOLUTION

2 (a)

$\vec{c} = \overrightarrow{oc} = \overrightarrow{ab} = \vec{b} - \vec{a} = (4\vec{i} + 3\vec{j}) - (3\vec{i} - \vec{j}) = \vec{i} + 4\vec{j}$



2 (b) (i)

2. $\theta = 90^\circ \Leftrightarrow \overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos 90^\circ = 0$
[If two vectors are perpendicular their dot product is zero. If their dot product is zero, the vectors are perpendicular.]

If $\vec{p} \perp \vec{q} \Rightarrow \vec{p}.\vec{q} = 0 \Rightarrow (2\vec{i} + \vec{j}).(3\vec{i} + k\vec{j}) = 0$
$\Rightarrow 6 + k = 0 \Rightarrow k = -6$

TRICK: Just multiply the $\vec{i}$ coefficients and multiply the $\vec{j}$ coefficients.

2 (b) (ii)

$\overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos\theta$ 8

$\overrightarrow{op}.\overrightarrow{or} = |\overrightarrow{op}||\overrightarrow{or}|\cos 45^\circ$

$\Rightarrow (2\vec{i}+\vec{j}).(3\vec{i}+t\vec{j}) = |2\vec{i}+\vec{j}||3\vec{i}+t\vec{j}|(\frac{1}{\sqrt{2}})$

$\vec{r} = x\vec{i}+y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2+y^2}$ 5

$\Rightarrow 6+t = \dfrac{\sqrt{5}\sqrt{9+t^2}}{\sqrt{2}} \Rightarrow \sqrt{2}(6+t) = \sqrt{5}\sqrt{9+t^2}$

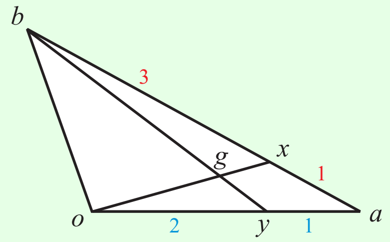
$\Rightarrow 2(6+t)^2 = 5(9+t^2) \Rightarrow 2(36+12t+t^2) = 45+5t^2$

$\Rightarrow 72+24t+2t^2 = 45+5t^2 \Rightarrow 3t^2-24t-27=0$

$\Rightarrow t^2-8t-9=0 \Rightarrow (t-9)(t+1)=0 \Rightarrow t=-1,\ 9$

2 (c) (i)

$\vec{c} = \dfrac{m\vec{b}+n\vec{a}}{m+n}$ 2



$\vec{x} = \dfrac{3\vec{a}+\vec{b}}{4} = \frac{3}{4}\vec{a}+\frac{1}{4}\vec{b}$

2 (c) (ii)

$\vec{y} = \frac{2}{3}\vec{a}$

$\overrightarrow{by} = \vec{y}-\vec{b} = \frac{2}{3}\vec{a}-\vec{b}$

2 (c) (iii)

$\vec{g} = m\vec{x} = \frac{3}{4}m\vec{a}+\frac{1}{4}m\vec{b}$ ....(**1**)

$\overrightarrow{bg} = n\overrightarrow{by} \Rightarrow \vec{g}-\vec{b} = \frac{2}{3}n\vec{a}-n\vec{b} \Rightarrow \vec{g} = \frac{2}{3}n\vec{a}+(1-n)\vec{b}$ ....(**2**)

Equating (**1**) and (**2**) $\Rightarrow \frac{3}{4}m\vec{a}+\frac{1}{4}m\vec{b} = \frac{2}{3}n\vec{a}+(1-n)\vec{b}$

Equate the $\vec{a}$ and $\vec{b}$ parts $\Rightarrow \frac{3}{4}m = \frac{2}{3}n$ ...(**1**) and $\frac{1}{4}m = (1-n)$ ...(**2**)

From equation (**1**): $\Rightarrow 9m = 8n \Rightarrow n = \frac{9}{8}m$

Substitute this value of n into equation (**2**): $\frac{1}{4}m = (1-n) \Rightarrow \frac{1}{4}m = 1-\frac{9}{8}m \Rightarrow \frac{11}{8}m = 1 \Rightarrow m = \frac{8}{11}$

$\Rightarrow n = \frac{9}{8}(\frac{8}{11}) = \frac{9}{11}$