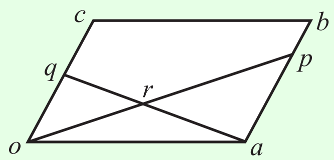
# VECTORS (Q 2, PAPER 2)

## 2002

2 (a) $\vec{s} = 4\vec{i} - 3\vec{j}$ and $\vec{t} = 2\vec{i} - 5\vec{j}$. Find $|\vec{st}|$.

2 (b) *oabc* is a parallelogram, where *o* is the origin. $p \in [ab]$ such that $|ap|:|pb| = 3:1$. *q* is the midpoint of [*oc*].

(i) Using equiangular triangles, or otherwise, find the ratio $|or|:|rp|$.




(ii) Express $\vec{p}$, and hence $\vec{r}$, in terms of $\vec{a}$ and $\vec{b}$.

2 (c) $\vec{k} = \vec{i} + 3\vec{j}$, $\vec{n} = 4\vec{i} - 2\vec{j}$, $\vec{u} = 2\vec{i} + \vec{j}$ and $\vec{v} = x\vec{i} + y\vec{j}$ where $x, y \in \mathbf{R}$.

(i) Express the value of $\vec{kn}.\vec{kv}$ in the form $ax + by + c$ where $a, b, c \in \mathbf{R}$.

(ii) Prove that if $\vec{kn}.\vec{kv} = \vec{kn}.\vec{ku}$, and $\vec{u} \neq \vec{v}$, then $\vec{kn} \perp \vec{uv}$.

**SOLUTION**

**2 (a)**

$|\vec{st}| = |\vec{t} - \vec{s}| = |(2\vec{i} - 5\vec{j}) - (4\vec{i} - 3\vec{j})| = |-2\vec{i} - 2\vec{j}|$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\vec{ab} = \vec{b} - \vec{a}$ ....... 1

$\vec{r} = x\vec{i} + y\vec{j} \Rightarrow |\vec{r}| = \sqrt{x^2 + y^2}$ ....... 5

**2 (b) (i)**

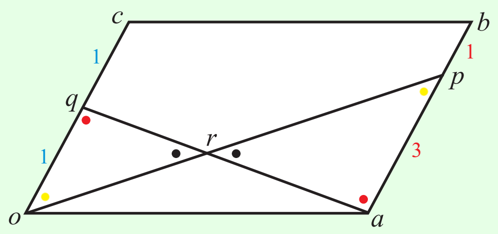
Consider triangles Δ *qro* and Δ *pra*.

$\angle qro = \angle pra$ (Vertically opposite angles)

$\angle rqo = \angle par$ (Alternate angles)

$\angle roq = \angle apr$ (Alternate angles)

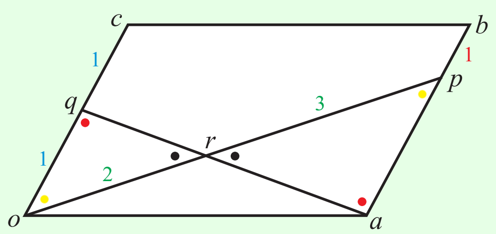
Therefore, triangles Δ *qro* and Δ *pra* are equi-angular. This means the ratio of their corresponding sides are equal.




$$\therefore \frac{|or|}{|rp|} = \frac{|oq|}{|ap|} = \frac{\frac{1}{2}|oc|}{\frac{3}{4}|ab|} = \frac{2|oc|}{3|oc|} = \frac{2}{3} \Rightarrow |or|:|rp| = 2:3$$

**2 (b) (ii)**

$\vec{c} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$ ....... 2




$$\vec{p} = \frac{\vec{a} + 3\vec{b}}{4} = \tfrac{1}{4}\vec{a} + \tfrac{3}{4}\vec{b}$$

$$\vec{r} = \tfrac{2}{5}\vec{p} = \tfrac{2}{5}(\tfrac{1}{4}\vec{a} + \tfrac{3}{4}\vec{b}) = \tfrac{1}{10}\vec{a} + \tfrac{3}{10}\vec{b}$$

**2 (c) (i)**

$\overrightarrow{kn}.\overrightarrow{kv} = (\vec{n} - \vec{k}).(\vec{v} - \vec{k}) = (4\vec{i} - 2\vec{j} - \vec{i} - 3\vec{j}).(x\vec{i} + y\vec{j} - \vec{i} - 3\vec{j})$

$= (3\vec{i} - 5\vec{j}).((x-1)\vec{i} + (y-3)\vec{j}) = 3(x-1) - 5(y-3)$

$= 3x - 3 - 5y + 15 = 3x - 5y + 12$

**2 (c) (ii)**

If $\overrightarrow{kn}.\overrightarrow{kv} = \overrightarrow{kn}.\overrightarrow{ku} \Rightarrow 3x - 5y + 12 = (\vec{n} - \vec{k}).(\vec{u} - \vec{k})$

$\Rightarrow 3x - 5y + 12 = (4\vec{i} - 2\vec{j} - \vec{i} - 3\vec{j}).(2\vec{i} + \vec{j} - \vec{i} - 3\vec{j})$

$\Rightarrow 3x - 5y + 12 = (3\vec{i} - 5\vec{j}).(\vec{i} - 2\vec{j}) = 13 \Rightarrow 3x - 5y - 1 = 0$ ....(**1**)

**TRICK**: Just multiply the $\vec{i}$ coefficients and multiply the $\vec{j}$ coefficients.

**2.** $\theta = 90^\circ \Leftrightarrow \overrightarrow{ab}.\overrightarrow{ac} = |\overrightarrow{ab}||\overrightarrow{ac}|\cos 90^\circ = 0$
[If two vectors are perpendicular their dot product is zero. If their dot product is zero, the vectors are perpendicular.]

You can prove that $\overrightarrow{kn} \perp \overrightarrow{uv}$ by showing that their dot product is zero.

$\overrightarrow{kn}.\overrightarrow{uv} = (\vec{n} - \vec{k}).(\vec{v} - \vec{u}) = (4\vec{i} - 2\vec{j} - \vec{i} - 3\vec{j}).(x\vec{i} + y\vec{j} - 2\vec{i} - \vec{j})$

$= (3\vec{i} - 5\vec{j}).((x-2)\vec{i} + (y-1)\vec{j}) = 3(x-2) - 5(y-1)$

$= 3x - 6 - 5y + 5 = 3x - 5y - 1 = 0$ (From equation (**1**)).