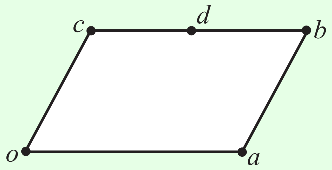
# VECTORS (Q 2, PAPER 2)

## 2001

2 (a) *oabc* is a parallelogram where *o* is the origin. *d* is the midpoint of [*cb*].

(i) Express $\vec{b}$ in terms of $\vec{a}$ and $\vec{c}$.

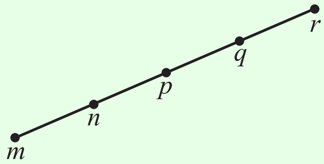
(ii) Express $\vec{d}$ in terms of $\vec{a}$ and $\vec{c}$.




2 (b) [*mr*] is divided into four line segments of equal length by the points *n*, *p* and *q*. Given that $\vec{m} = -2\vec{i} + 3\vec{j}$ and $\vec{q} = 7\vec{i} - 9\vec{j}$, express

(i) $\vec{p}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.




2 (c) *rst* is a triangle where $\vec{r} = -\vec{i} + 2\vec{j}$, $\vec{s} = -4\vec{i} - 2\vec{j}$ and $\vec{t} = 3\vec{i} - \vec{j}$.

(i) Express $\overrightarrow{rs}$, $\overrightarrow{st}$ and $\overrightarrow{tr}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Show that the triangle *rst* is right-angled at *r*.

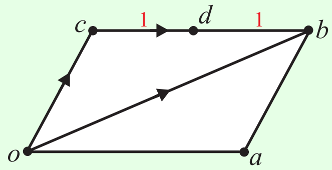
(iii) Find the measure of $\angle rst$.

**SOLUTION**

**2 (a) (i)**

$\vec{b} = \vec{c} + \overrightarrow{cb}$

However, $\vec{a} = \overrightarrow{cb} \Rightarrow \vec{b} = \vec{a} + \vec{c}$

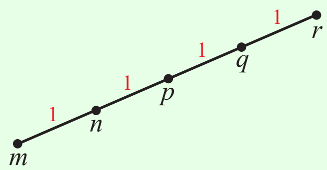



**2 (a) (ii)**

$$\vec{d} = \frac{\vec{b} + \vec{c}}{2} = \frac{\vec{a} + \vec{c} + \vec{c}}{2} = \tfrac{1}{2}\vec{a} + \vec{c}$$

$$\vec{c} = \frac{m\vec{b} + n\vec{a}}{m+n}$$ ....... 2

**2 (b) (i)**




$$\vec{p} = \frac{\vec{m} + 2\vec{q}}{3} = \frac{(-2\vec{i} + 3\vec{j}) + 2(7\vec{i} - 9\vec{j})}{3}$$

$$= \frac{12\vec{i} - 15\vec{j}}{3} = 4\vec{i} - 5\vec{j}$$

**2 (b) (ii)**

$$\vec{q} = \frac{3\vec{r} + \vec{m}}{4} \Rightarrow \vec{r} = \frac{4\vec{q} - m}{3} = \frac{4(7\vec{i} - 9\vec{j}) - (-2\vec{i} + 3\vec{j})}{3}$$

$$= \frac{30\vec{i} - 39\vec{j}}{3} = 10\vec{i} - 13\vec{j}$$

**2 (c) (i)**

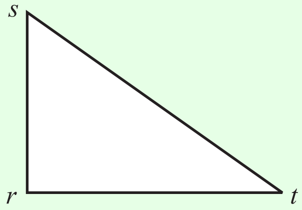
$\vec{rs} = \vec{s} - \vec{r} = -4\vec{i} - 2\vec{j} + \vec{i} - 2\vec{j} = -3\vec{i} - 4\vec{j}$

$\vec{st} = \vec{t} - \vec{s} = 3\vec{i} - \vec{j} + 4\vec{i} + 2\vec{j} = 7\vec{i} + \vec{j}$ $\quad$ $\vec{ab} = \vec{b} - \vec{a}$ ....... **1**

$\vec{tr} = \vec{r} - \vec{t} = -\vec{i} + 2\vec{j} - 3\vec{i} + \vec{j} = -4\vec{i} + 3\vec{j}$

**2 (c) (ii)**

**2.** $\theta = 90^\circ \Leftrightarrow \vec{ab}.\vec{ac} = |\vec{ab}||\vec{ac}|\cos 90^\circ = 0$
[If two vectors are perpendicular their dot product is zero. If their dot product is zero, the vectors are perpendicular.]




TRICK: Just multiply the $\vec{i}$ coefficients and multiply the $\vec{j}$ coefficients when doing the dot product..

You need to show that $\vec{sr}.\vec{rt} = 0$.

$\vec{rs}.\vec{tr} = (-3\vec{i} - 4\vec{j}).(-4\vec{i} + 3\vec{j}) = 12 - 12 = 0$

**2 (c) (iii)**

$\vec{ab}.\vec{ac} = |\vec{ab}||\vec{ac}|\cos\theta$ ....... **8**

$$\vec{sr}.\vec{st} = |\vec{sr}||\vec{st}|\cos\angle rst \Rightarrow \cos\angle rst = \frac{\vec{sr}.\vec{st}}{|\vec{sr}||\vec{st}|}$$

$$\Rightarrow \cos\angle rst = \frac{(3\vec{i} + 4\vec{j}).(7\vec{i} + \vec{j})}{|3\vec{i} + 4\vec{j}||7\vec{i} + \vec{j}|} \Rightarrow \cos\angle rst = \frac{21+4}{\sqrt{25}\sqrt{50}} = \frac{25}{5\times 5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \angle rst = 45^\circ$$