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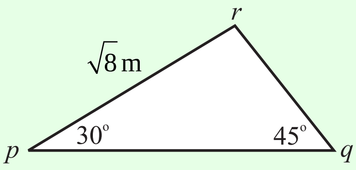
4 (a) Show that the area of the triangle pqr, correct to one decimal place, is 2.7 m^2, if $|pr| = \sqrt{8}$ m, $|\angle rpq| = 30^\circ$ and $|\angle pqr| = 45^\circ$.



(b) If $\tan(45^\circ - A) = \dfrac{1-\tan A}{1+\tan A}$ show that $\dfrac{\cos 2A}{1+\sin 2A} = \tan(45^\circ - A)$.

Deduce that $\tan 22\tfrac{1}{2}^\circ = \dfrac{1}{\sqrt{2}+1}$.

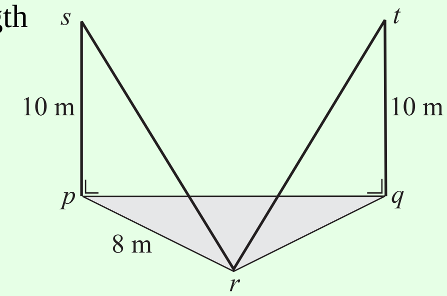
(c) [sp], [tq] are vertical poles each of height 10 m, p, q, r are points on level ground. Two wires of equal length join s and t to r, i.e. $|sr| = |st|$.

If $|pr| = 8$ m, $|\angle pqr| = 32^\circ 12'$, $|\angle prq| = 120^\circ$, calculate

(i) $|pq|$ to the nearest metre

(ii) $|sr|$ in surd form

(iii) $|\angle srt|$ to the nearest degree.

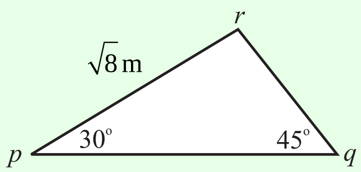


SOLUTION

4 (a)

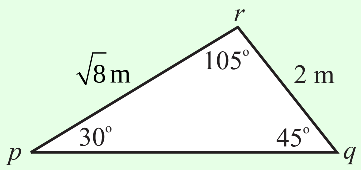
Use the Sine rule to find $|rq|$ first.

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$ **18**



$$\therefore \frac{|rq|}{\sin 30^\circ} = \frac{\sqrt{8}}{\sin 45^\circ} \Rightarrow |rq| = \frac{\sqrt{8}\sin 30^\circ}{\sin 45^\circ} = 2$$

$$\boxed{A = \tfrac{1}{2}ab\sin C}$$ **5**



$$A = \tfrac{1}{2}(2)(\sqrt{8})\sin 105^\circ = 2.7 \text{ m}^2$$

4 (b)

Steps

1. You have to prove that the left-hand side (*LHS*) equals the right-hand side (*RHS*).
2. Change everything to sine and cosine.
3. Simplify each side using page 9 of the tables and good algebra.
4. For half angles, $\frac{\theta}{2}$: Let $\frac{\theta}{2} = A \Rightarrow \theta = 2A$.

LHS

$$\frac{\cos 2A}{1+\sin 2A}$$

$$= \frac{\cos^2 - \sin^2 A}{1+2\sin A\cos A}$$

RHS

$$\frac{1-\tan A}{1+\tan A}$$

$$= \frac{(1-\frac{\sin A}{\cos A})}{(1+\frac{\sin A}{\cos A})} \times \frac{\cos A}{\cos A}$$

$$= \frac{\cos A - \sin A}{\cos A + \sin A}$$

$$= \frac{(\cos A - \sin A)}{(\cos A + \sin A)} \times \frac{(\cos A + \sin A)}{(\cos A + \sin A)}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos^2 A + 2\sin A\cos A + \sin^2 A}$$

$$= \frac{\cos^2 - \sin^2 A}{1+2\sin A\cos A}$$

You have prove that $\frac{\cos 2A}{1+\sin 2A} = \tan(45^o - A)$. Let $A = 22\frac{1}{2}^o$.

$$\Rightarrow \tan(45^o - 22\tfrac{1}{2}^o) = \frac{\cos 2(22\frac{1}{2}^o)}{1+\sin 2(22\frac{1}{2}^o)} \Rightarrow \tan 22\tfrac{1}{2}^o = \frac{\cos 45^o}{1+\sin 45^o}$$

$$\Rightarrow \tan 22\tfrac{1}{2}^o = \frac{\frac{1}{\sqrt{2}}}{(1+\frac{1}{\sqrt{2}})} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}+1}$$

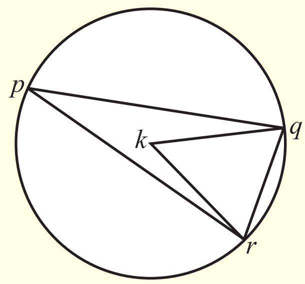
4 (c)

There is an error is this question so don't bother doing it.

5 (a) Find the value k, if

$$k=\frac{\cos(\frac{\pi}{4}+\theta)-\cos(\frac{\pi}{4}-\theta)}{\sin(\frac{\pi}{4}+\theta)-\sin(\frac{\pi}{4}-\theta)} \text{ where } \sin\theta\neq 0.$$

(b) p, q, r are points of a circle, centre k. The length of the radius of the circle is 2 cm.



The length of the minor arc pq is $\frac{5\pi}{3}$ cm.

(i) Find the length of the chord $[pq]$, correct to two places of decimals.

(ii) If $|pq| = |pr|$, find $|rq|$.

(c) $x = 0^\circ$ and $x = 60^\circ$ are two solutions of the equation $a\sin^2 2x+\cos 2x-b=0$ where $a,b\in\mathbf{N}$.
Find the value of a and the value of b.
Using these values of a and b, find all the solutions of the equation where $0^\circ\leq x\leq 360^\circ$.

Solution

5 (a)

$$k=\frac{\cos(\frac{\pi}{4}+\theta)-\cos(\frac{\pi}{4}-\theta)}{\sin(\frac{\pi}{4}+\theta)-\sin(\frac{\pi}{4}-\theta)}=\frac{-2\sin(\frac{\pi}{4})\sin\theta}{2\cos(\frac{\pi}{4})\sin\theta}$$

$$=\tan(\tfrac{\pi}{4})=-1$$

$$\tan A=\frac{\sin A}{\cos A}$$

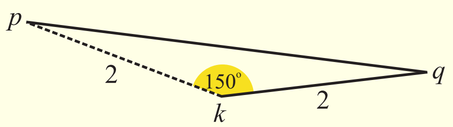
Sums $\longrightarrow$ Products
$\sin A+\sin B=2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$
$\sin A-\sin B=2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$
$\cos A+\cos B=2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$
$\cos A-\cos B=-2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$

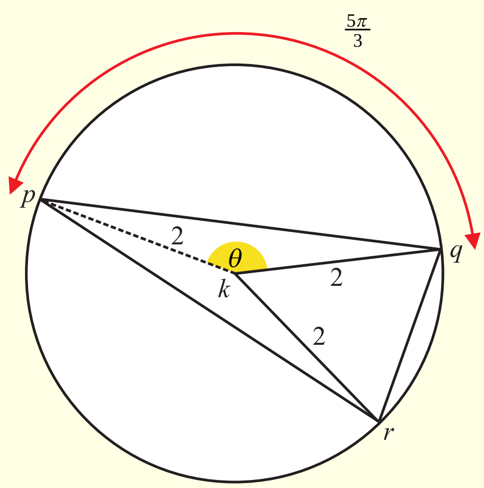
5 (b) (i)

$$\theta=\frac{s}{r}=\frac{\frac{5\pi}{3}}{2}=\frac{5\pi}{6}=150^\circ$$

Arc length s

$s=r\theta$ **6**





The Cosine Rule

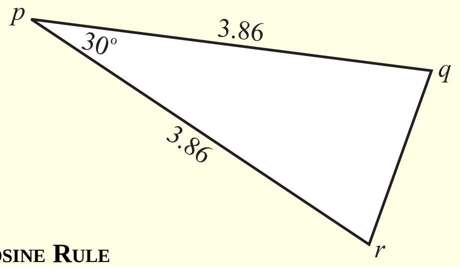
$a^2=b^2+c^2-2bc\cos A$ **19**

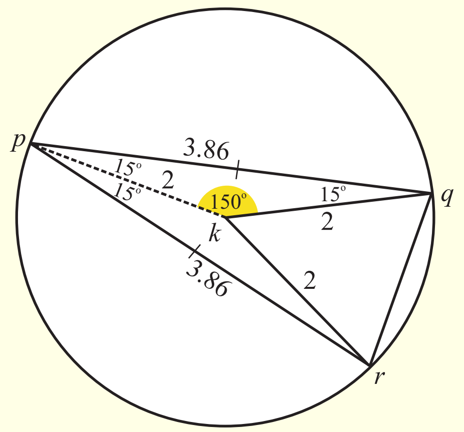
$$|pq|^2=2^2+2^2-2(2)(2)\cos 150^\circ\Rightarrow|pq|=3.86\text{ cm}$$

5 (b) (ii)

You can see from the diagram that $|\angle qpr| = 30^\circ$.

Lift out triangle *qpr*.





The Cosine Rule

$a^2 = b^2 + c^2 - 2bc\cos A$ **19**

$$|rq|^2 = 3.86^2 + 3.86^2 - 2(3.86)(3.86)\cos 30^\circ \Rightarrow |rq| = 2 \text{ cm}$$

5 (c)

As $x = 0^\circ$ and $x = 60^\circ$ are two solutions, they satisfy the equation so you can substitute them into the equation.

$x = 0^\circ$: $\quad a\sin^2 0^\circ + \cos 0^\circ - b = 0 \Rightarrow 1 - b = 0 \Rightarrow b = 1$

$x = 60^\circ$: $\quad a\sin^2 120^\circ + \cos 120^\circ - 1 = 0 \Rightarrow a(\frac{\sqrt{3}}{2})^2 - \frac{1}{2} - 1 = 0$

$\Rightarrow a(\frac{3}{4}) - \frac{3}{2} = 0 \Rightarrow a(\frac{3}{4}) = \frac{3}{2} \Rightarrow a = 2$

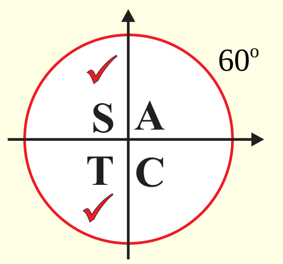
Solve $2\sin^2 2x + \cos 2x - 1 = 0$. $\quad \cos^2 A + \sin^2 A = 1$ **8**

$\therefore 2(1 - \cos^2 2x) + \cos 2x - 1 = 0 \Rightarrow 2 - 2\cos^2 2x + \cos 2x - 1 = 0$

$\Rightarrow 2\cos^2 2x - \cos 2x - 1 = 0$

$\Rightarrow (2\cos 2x + 1)(\cos 2x - 1) = 0$

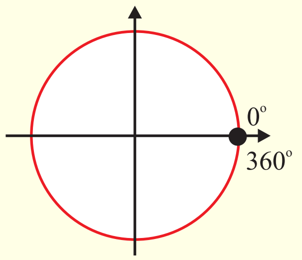
$\therefore \cos 2x = -\frac{1}{2}$



$2x$ = 120°, 480° (Second quadrant)
= 240°, 600° (Third quadrant)

x = 60°, 240° (Second quadrant)
= 120°, 300° (Third quadrant)

$\therefore \cos 2x = 1$



$2x$ = 0°, 360°, 720°

x = 0°, 180°, 360°

x = 0°, 60°, 120°, 180°, 240°, 300°, 360°