Trigonometry (Q 4 & 5, Paper 2)

Lesson No. 7: Trig Equations

2001

4 (b) (i) Write cos 2x in terms of sin x.

(ii) Hence, find all the solutions of the equation $\cos 2x - \sin x = 1$ in the domain $0^\circ \le x \le 360^\circ$.

4 (b) (i)

$\cos 2x = \cos^2 x - \sin^2 x$

$= (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$

$\cos 2A = \cos^2 A - \sin^2 A$ **14**

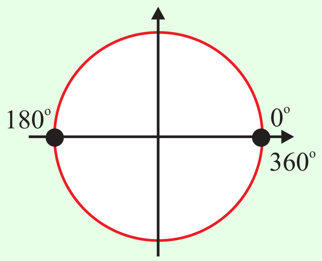
$\cos^2 A + \sin^2 A = 1$ **8**

4 (b) (ii)

$\cos 2x - \sin x = 1 \Rightarrow 1 - 2\sin^2 x - \sin x - 1 = 0$

$\Rightarrow 2\sin^2 x + \sin x = 0 \Rightarrow \sin x(2\sin x + 1) = 0$

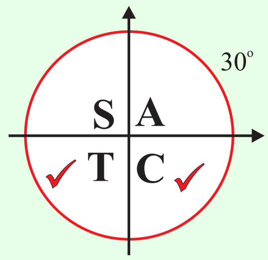
$\Rightarrow \sin x = 0$



$x = 0^\circ, 360^\circ$

$= 180^\circ$

$\Rightarrow \sin x = -\frac{1}{2}$



$x = 210^\circ$ [Third quadrant]

$= 330^\circ$ [Fourth quadrant]

Ans: 0°, 180°, 210°, 330°, 360°

2002

4 (a) Find the value of θ for which $\cos\theta = -\frac{\sqrt{3}}{2}$, $0^\circ \le \theta \le 180^\circ$.

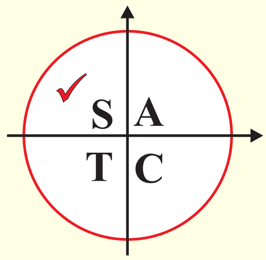
4 (b) (i) Use the formula $\sin^2 A = \frac{1}{2}(1-\cos 2A)$ to express $\sin^2 \frac{1}{2}x$ in terms of cos x.

(ii) Hence, or otherwise, find all the solutions of the equation $\sin^2 \frac{1}{2}x - \cos^2 x = 0$ in the domain $0^\circ \le \theta \le 360^\circ$.

Solution

4 (a)

The angle θ is in the second quadrant as cos is negative in this quadrant and it is within the range specified.
Find the basic angle from page 9 of the tables.



$\cos^{-1}(\frac{\sqrt{3}}{2}) = 30^\circ$

$\therefore \theta = 180^\circ - 30^\circ = 150^\circ$

4 (b) (i)

$\sin^2 A = \frac{1}{2}(1-\cos 2A)$

Let $A = \frac{1}{2}x \Rightarrow 2A = x$

$\therefore \sin^2 \frac{1}{2}x = \frac{1}{2}(1-\cos x)$

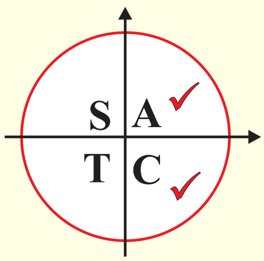
4 (b) (ii)

$\sin^2 \frac{1}{2}x - \cos^2 x = 0 \Rightarrow \frac{1}{2}(1-\cos x) - \cos^2 x = 0$

$\Rightarrow 1-\cos x - 2\cos^2 x = 0 \Rightarrow 2\cos^2 x + \cos x - 1 = 0$

$\Rightarrow (2\cos x - 1)(\cos x + 1) = 0$

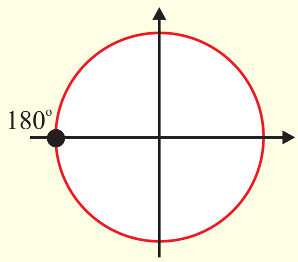
$\Rightarrow \cos x = \frac{1}{2}$



$x = 60^\circ$

$= 300^\circ$

$\Rightarrow \cos x = -1$



$x = 180^\circ$

Ans: 60°, 180°, 300°

2003

4 (b) Find all the solutions of the equation $\sin 2x + \sin x = 0$ in the domain $0^\circ \le x \le 360^\circ$.

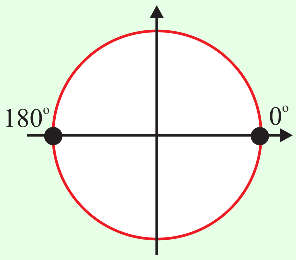
SOLUTION

4 (b)

$\sin 2x + \sin x = 0 \Rightarrow 2\sin x \cos x + \sin x = 0$

$\Rightarrow \sin x(2\cos x + 1) = 0$

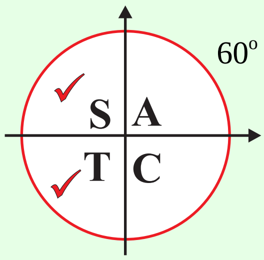
$\Rightarrow \sin x = 0$



$x = 0^\circ, 360^\circ$

$= 180^\circ$

$\Rightarrow \cos x = -\frac{1}{2}$



$x = 120^\circ$ [Second quadrant]

$= 240^\circ$ [Third quadrant]

Ans: $0^\circ, 120^\circ, 180^\circ, 240^\circ, 360^\circ$

2004

4 (b) (i) Prove that $\cos 2A = \cos^2 A - \sin^2 A$. Deduce that $\cos 2A = 2\cos^2 A - 1$.

(ii) Hence, or otherwise, find the value of θ for which $2\cos\theta - 7\cos(\frac{\theta}{2}) = 0$, where $0^\circ \le \theta \le 360^\circ$. Give your answer correct to the nearest degree.

SOLUTION

4 (b) (ii)

Half angles are the hardest with which to deal.
For a half angle: Let $\frac{x}{2} = A \Rightarrow x = 2A$

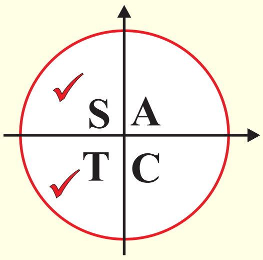
Let $\frac{\theta}{2} = A \Rightarrow \theta = 2A$

$2\cos\theta - 7\cos(\frac{\theta}{2}) = 0 \Rightarrow 2\cos 2A - 7\cos A = 0$

Use the previous result: $\therefore 2(2\cos^2 A - 1) - 7\cos A = 0 \Rightarrow 4\cos^2 A - 7\cos A - 2 = 0$

$\Rightarrow (4\cos A + 1)(\cos A - 2) = 0$

$\Rightarrow \cos A = -\frac{1}{4}$



$\cos^{-1}(0\cdot 25) = 75\cdot 52^\circ$

$A = 104\cdot 48^\circ$ [Second quadrant]

$= 255\cdot 52^\circ$ [Third quadrant]

$\theta = 2A = 209^\circ$

$= 511^\circ$ [Outside the required range]

$\Rightarrow \cos A = 2$

If you look up the inverse cos of 2 on your calculator you will get an error reading. There are no solutions to this equation.

Ans: 209°

2005

4 (b) (i) Using $\cos 2A - \cos^2 A - \sin^2 A$, or otherwise, prove $\cos^2 A = \frac{1}{2}(1+\cos 2A)$.

(ii) Hence, or otherwise, solve the equation $1+\cos 2x = \cos x$, where $0^\circ \le x \le 360^\circ$.

SOLUTION

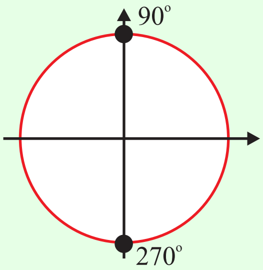
4 (b) (ii)

Solve $1+\cos 2x = \cos x$.

Use the previous result: $\cos^2 x = \frac{1}{2}(1+\cos 2x) \Rightarrow 2\cos^2 x = 1+\cos 2x$

$\therefore 2\cos^2 x = \cos x \Rightarrow 2\cos^2 x - \cos x = 0 \Rightarrow \cos x(2\cos x - 1) = 0$

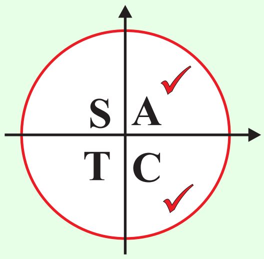
$\Rightarrow \cos x = 0$



$x = 90^\circ$

$= 270^\circ$

$\Rightarrow \cos x = \frac{1}{2}$



$x = 60^\circ$

$= 300^\circ$

Ans: $60^\circ, 90^\circ, 270^\circ, 300^\circ$

2006

4 (a) Write down the values of A for which $\cos A = \frac{1}{2}$, where $0^\circ \le A \le 360^\circ$.

4 (b) (i) Express $\sin(3x + 60^\circ) - \sin x$ as a product of sine and cosine.

(ii) Find all the solutions of the equation $\sin(3x + 60^\circ) - \sin x = 0$, where $0^\circ \le A \le 360^\circ$.

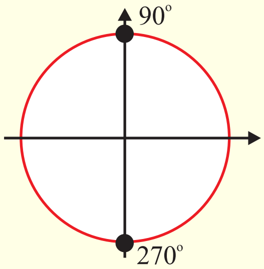
SOLUTION

4 (b) (ii)

$\sin(3x + 60^\circ) - \sin x = 0 \Rightarrow 2\cos(2x + 30^\circ)\sin(x + 30^\circ) = 0$

$\Rightarrow \cos(2x + 30^\circ)\sin(x + 30^\circ) = 0$

$\Rightarrow \cos(2x + 30^\circ) = 0$



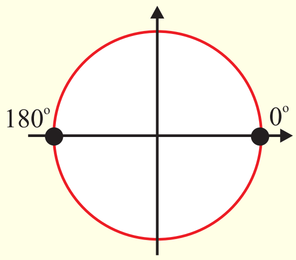
$2x + 30^\circ = 90^\circ, 450^\circ$

$= 270^\circ, 630^\circ$

$x = 30^\circ, 210^\circ$

$= 120^\circ, 300^\circ$

$\Rightarrow \sin(x + 30^\circ) = 0$



$x + 30^\circ = 0^\circ, 360^\circ$

$= 180^\circ$

$x = 330^\circ$

$= 150^\circ$

Ans: $30^\circ, 120^\circ, 150^\circ, 210^\circ, 300^\circ, 330^\circ$