TRIGONOMETRY (Q 4 & 5, PAPER 2)

LESSON NO. 5: TRIG IDENTITIES

2005

4 (b) (i) Using $\cos 2A - \cos^2 A - \sin^2 A$, or otherwise, prove $\cos^2 A = \frac{1}{2}(1+\cos 2A)$.

SOLUTION

4 (b) (i)

STEPS

1. You have to prove that the left-hand side (*LHS*) equals the right-hand side (*RHS*).
2. Change everything to sine and cosine.
3. Simplify each side using page 9 of the tables and good algebra.
4. For half angles, $\frac{\theta}{2}$: Let $\frac{\theta}{2} = A \Rightarrow \theta = 2A$.

LHS	RHS
$\cos^2 A$	$\frac{1}{2}(1+\cos 2A)$
	$= \frac{1}{2}(1+\cos^2 A - \sin^2 A)$
	$= \frac{1}{2}(\cos^2 A + \cos^2 A)$ [Using $\cos^2 A + \sin^2 A = 1$]
	$= \frac{1}{2}(2\cos^2 A) = \cos^2 A$

LHS = RHS

2004

4 (b) (i) Prove that $\cos 2A = \cos^2 A - \sin^2 A$. Deduce that $\cos 2A = 2\cos^2 A - 1$.

5 (b) (i) Show that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2$ simplifies to a constant.

(ii) Express $1-(\cos x - \sin x)^2$ in the form $a \sin bx$, where $a, b \in \mathbf{Z}$.

SOLUTION

4 (b) (i)

Use formula **11** from page 9 of the tables and replace B by A.

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\Rightarrow \cos(A+A) = \cos A \cos A - \sin A \sin A$

$\Rightarrow \cos 2A = \cos^2 A - \sin^2 A$

Using formula 8 from page 9 of the tables:

$\cos^2 A + \sin^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A$

$\therefore \cos 2A = \cos^2 A - (1-\cos^2 A)$

$\Rightarrow \cos 2A = \cos^2 A - 1 + \cos^2 A \Rightarrow \cos 2A = 2\cos^2 A - 1$

COMPOUND ANGLES	
$\sin(A+B) = \sin A \cos B + \cos A \sin B$	 9
$\sin(A-B) = \sin A \cos B - \cos A \sin B$	 10
$\cos(A+B) = \cos A \cos B - \sin A \sin B$	 11
$\cos(A-B) = \cos A \cos B + \sin A \sin B$	 12

$\cos^2 A + \sin^2 A = 1$ 8

CONT...

5 (b) (i)

$(\cos x+\sin x)^2+(\cos x-\sin x)^2=\cos^2 x+2\cos x\sin x+\sin^2 x+\cos^2 x-2\cos x\sin x+\sin^2 x$

$=\cos^2 x+\sin^2 x+\cos^2 x+\sin^2 x=1+1=2$ $\boxed{\cos^2 A+\sin^2 A=1}$ **8**

5 (b) (ii)

$1-(\cos x-\sin x)^2=1-(\cos^2 x-2\cos x\sin x+\sin^2 x)$

$=1-(1-2\cos x\sin x)=2\cos x\sin x=\sin 2x$ $\boxed{\sin 2A=2\sin A\cos A}$ **13**

2002

5 (b) (i) Prove that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$.

Solution

5 (b) (i)

This is a trig identity.

Steps

1. You have to prove that the left-hand side (*LHS*) equals the right-hand side (*RHS*).
2. Change everything to sine and cosine.
3. Simplify each side using page 9 of the tables and good algebra.
4. For half angles, $\frac{\theta}{2}$: Let $\frac{\theta}{2}=A\Rightarrow\theta=2A$.

LHS

$$\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}$$

$$=\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}$$

RHS

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}{1-(\frac{\sin A}{\cos A})(\frac{\sin B}{\cos B})}$$

$$=\frac{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}{1-(\frac{\sin A}{\cos A})(\frac{\sin B}{\cos B})}\times\frac{\cos A\cos B}{\cos A\cos B}$$

$$=\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}$$

LHS = *RHS*

2001

5 (c) A is an obtuse angle such that $\sin\left(A+\frac{\pi}{6}\right)+\sin\left(A-\frac{\pi}{6}\right)=\frac{4\sqrt{3}}{5}$.

(i) Find $\sin A$ and $\tan A$.

(ii) Given that $\tan(A+B)=\frac{1}{2}$, find $\tan B$ and express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbf{Z}$ and $q \neq 0$.

Solution

5 (c)

A is obtuse which means it is in the second quadrant.

5 (c) (i)

Compound Angles
$\sin(A+B)=\sin A\cos B+\cos A\sin B$ 9
$\sin(A-B)=\sin A\cos B-\cos A\sin B$ 10

$$\sin(A+30^\circ)+\sin(A-30^\circ)=\frac{4\sqrt{3}}{5}$$

$$\Rightarrow \sin A\cos 30^\circ+\sin 30^\circ\cos A+\sin A\cos 30^\circ-\sin 30^\circ\cos A=\frac{4\sqrt{3}}{5}$$

$$\Rightarrow 2\sin A\cos 30^\circ=\frac{4\sqrt{3}}{5}\Rightarrow 2\sin A\times\frac{\sqrt{3}}{2}=\frac{4\sqrt{3}}{5}$$

$$\Rightarrow \sin A=\tfrac{4}{5}$$

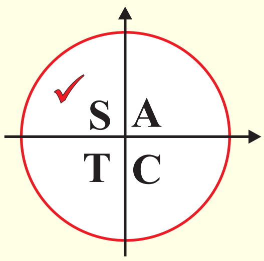
Draw a right-angled triangle and use Pythagoras to find the length of the third side.

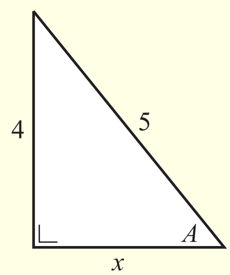
$$x^2+4^2=5^2\Rightarrow x=3$$

$\sin\theta=\frac{y}{r}=\frac{\text{Opposite}}{\text{Hypotenuse}}$ 2

$\tan\theta=\frac{y}{x}=\frac{\text{Opposite}}{\text{Adjacent}}$ 3

$x^2+y^2=r^2$ 4





You can see that tan is negative in the second quadrant.

$\therefore \tan A=-\frac{4}{3}$

5 (c) (ii)

$\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ 15

$$\tan(A+B)=\frac{1}{2}\Rightarrow\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{1}{2}$$

$$\Rightarrow 2\tan A+2\tan B=1-\tan A\tan B$$

$$\Rightarrow 2(-\tfrac{4}{3})+2\tan B=1-(-\tfrac{4}{3})\tan B$$

$$\Rightarrow -8+6\tan B=3+4\tan B$$

$$\Rightarrow 2\tan B=11\Rightarrow \tan B=\tfrac{11}{2}$$