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TRIGONOMETRY (Q 4 & 5, PAPER 2)

LESSON NO. 4: SINGLE AND COMPOUND ANGLES

2004

4 (a) A is an acute angle such that $\tan A = \frac{8}{15}$.

Without evaluating A, find

(i) $\cos A$

(ii) $\sin 2A$.

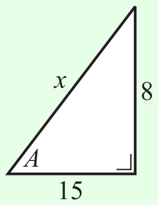
SOLUTION

4 (a)

Using Pythagoras:

$x^2 = 8^2 + 15^2 \Rightarrow x^2 = 64 + 225$

$\Rightarrow x^2 = 289 \Rightarrow x = 17$



$\cos\theta = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ 1

$\tan\theta = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}$ 3

$x^2 + y^2 = r^2$ 4

$\sin 2A = 2\sin A\cos A$ 13

4 (a) (i)

$\cos A = \frac{15}{17}$

4 (a) (ii)

$\sin 2A = 2\sin A\cos A = 2 \times \frac{8}{17} \times \frac{15}{17} = \frac{240}{289}$

2003

5 (a) Find the value of $\sin 15^\circ$ in surd form.

5 (c) (i) Using the identity $\cos(A-B) = \cos A\cos B + \sin A\sin B$, or otherwise, prove:

$\sin(A+B) = \sin A\cos B + \cos A\sin B$.

(ii) Prove: $\sin(A+B)\sin(A-B) = (\sin A + \sin B)(\sin A - \sin B)$.

SOLUTION

5 (a)

$\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

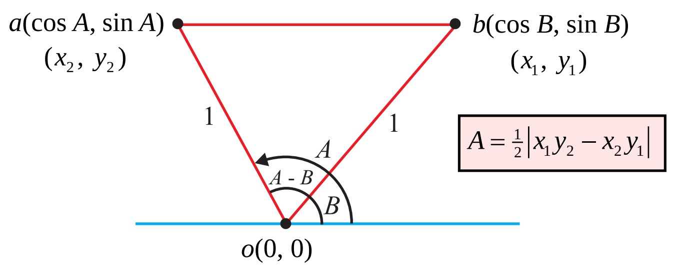
A	0	π	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	[Radians]
A	0°	180°	90°	60°	45°	30°	[Degrees]
$\cos A$	1	-1	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	
$\sin A$	0	0	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	
$\tan A$	0	0	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	

$\sin(A-B) = \sin A\cos B - \cos A\sin B$ 10

5 (c) (i)

The method for proving compound angle formulae involves using the diagram below to prove the formula for $\sin(A-B)$ and then derive the other formulae from this one.

PROOF: $\sin(A-B)=\sin A\cos B-\cos A\sin B$



Area $\Delta oba=\frac{1}{2}\times1\times1\times\sin(A-B)=\frac{1}{2}(\cos B\sin A-\sin B\cos A)$

$\Rightarrow \sin(A-B)=\sin A\cos B-\cos A\sin B$ .... Formula **10**

Replacing B by $-B$ in Formula **10**

$\Rightarrow \sin(A+B)=\sin A\cos(-B)-\cos A\sin(-B)$

$\Rightarrow \sin(A+B)=\sin A\cos B+\cos A\sin B$ .... Formula **9**

Using another method as directed by the question:

You are told that $\cos(A-B)=\cos A\cos B+\sin A\sin B.$

Replace A by $90^\circ-A$: $\therefore \cos((90^\circ-A)-B)=\cos(90^\circ-A)\cos B+\sin(90^\circ-A)\sin B$

$\Rightarrow \cos(90^\circ-(A+B))=\sin A\cos B+\cos A\sin B$

$\Rightarrow \sin(A+B)=\sin A\cos B+\cos A\sin B$

5 (c) (ii)

This is a trig identity.

STEPS

1. You have to prove that the left-hand side (*LHS*) equals the right-hand side (*RHS*).
2. Change everything to sine and cosine.
3. Simplify each side using page 9 of the tables and good algebra.
4. For half angles, $\frac{\theta}{2}$: Let $\frac{\theta}{2}=A\Rightarrow\theta=2A.$

LHS

$\sin(A+B)\sin(A-B)$

$=(\sin A\cos B+\cos A\sin B)(\sin A\cos B-\cos A\sin B)$

$=\sin^2 A\cos^2 B+\cos A\sin B\sin A\cos B-\cos A\sin B\sin A\cos B-\cos^2 A\sin^2 B$

$=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B$

$=\sin^2 A(1-\sin^2 B)-(1-\sin^2 A)\sin^2 B$

$=\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B-\sin^2 A\sin^2 B$

$=\sin^2 A-\sin^2 B=(\sin A+\sin B)(\sin A-\sin B)$

$=$ *RHS*

2002

5 (b) (ii) Prove that $\tan 22\tfrac{1}{2}^\circ = \sqrt{2} - 1$.

Solution

5 (b) (ii)

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ **17**

Let $A = 22\tfrac{1}{2}^\circ \Rightarrow 2A = 45^\circ$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} \Rightarrow \tan 45^\circ = \frac{2\tan A}{1-\tan^2 A}$$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ **4**

$$\Rightarrow 1 = \frac{2\tan A}{1-\tan^2 A} \Rightarrow 1-\tan^2 A = 2\tan A$$

$$\Rightarrow \tan^2 A + 2\tan A - 1 = 0$$

This is a quadratic equation that can be solved using formula **4**.

$a = 1,\ b = 2,\ c = -1$

$$\therefore \tan A = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)} = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm \sqrt{8}}{2}$$

$$\Rightarrow \tan A = \frac{-2 \pm 2\sqrt{2}}{2} \Rightarrow \tan A = -1 \pm \sqrt{2}$$

$\tan A$ is in the first quadrant which means it is positive.

$\therefore \tan A = \sqrt{2} - 1$