TRIGONOMETRY (Q 4 & 5, PAPER 2)

LESSON NO. 3: CIRCLES

2001

4 (a) The length of an arc of a circle is 10 cm. The radius of the circle is 4 cm. The measure of the angle at the centre of the circle subtended by the arc is θ.

(i) Find θ in radians.

(ii) Find θ in degrees, correct to the nearest degree.

SOLUTION

4 (a) (i)

$10 = 4\theta \Rightarrow \theta = 2\cdot 5$ rad

4 (a) (ii)

$$2\cdot 5 \text{ rad} = 2\cdot 5 \times \frac{180^\circ}{\pi} = 143^\circ$$

Arc length *s*

$s = r\theta$ **6**

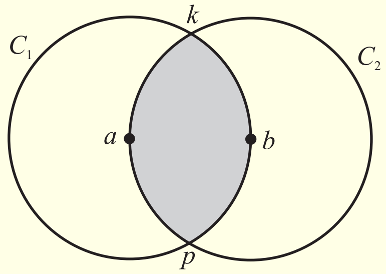
Degrees to radians: $\times \frac{\pi}{180^\circ}$

Radians to degrees: $\times \frac{180^\circ}{\pi}$

2003

4 (a) The circumference of a circle is 30π cm. The area of a sector of the circle is 75 cm^2. Find, in radians, the angle in this sector.

4 (c) C_1 is a circle with centre a and radius r. C_2 is a circle with centre b and radius r. C_1 and C_2 intersect at k and p. $a \in C_2$. $b \in C_1$.



(i) Find, in radians, the measure of angle kap.

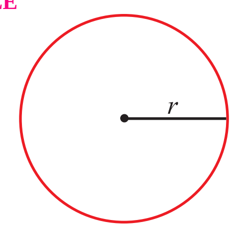
(ii) Calculate the area of the shaded region. Give your answer in terms of r and π.

SOLUTION

4 (a)

Use page 6 & 7 of the tables:

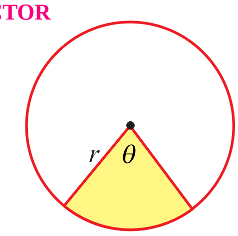
CIRCLE



Length $= 2\pi r$

Area $= \pi r^2$

SECTOR



Length $= r\theta\,(\theta$ in radians)

Area $= \frac{1}{2}r^2\theta\,(\theta$ in radians)

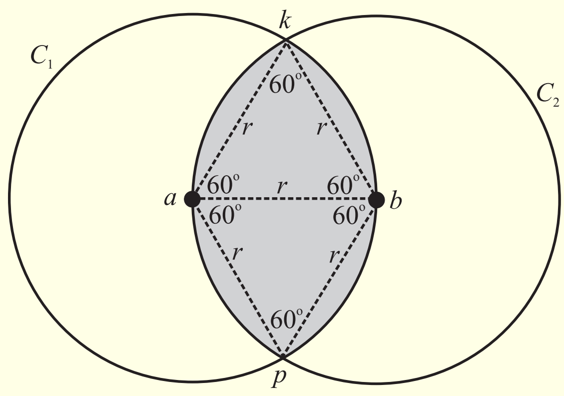
$2\pi r = 30\pi \Rightarrow r = 15$ cm

$$A = \tfrac{1}{2}(15)^2\theta = 75 \Rightarrow \theta = \frac{2\times 75}{225} = \tfrac{2}{3}\text{ rad}$$

4 (c) (i)

Δakb and Δpab are equilateral as all sides are equal to the radius r. Therefore, all angles are 60°.

$\angle kap = 120^\circ = \frac{2\pi}{3}$



4 (c) (ii)

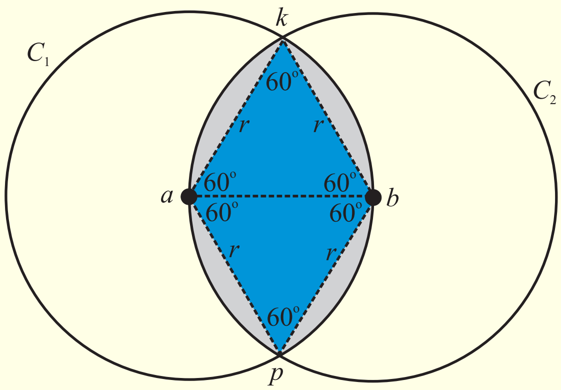
Area of the shaded region is the area of the 2 blue triangles plus the 4 grey segments.

To find the area of a grey segment:

Area of sector *kab* – Area of Δkab

$= \frac{1}{2}r^2(\frac{\pi}{3}) - \frac{1}{2}(r)(r)\sin 60^{\circ}$

$= (\frac{\pi}{6})r^2 - \frac{\sqrt{3}}{4}r^2$



Area of shaded region:

$A = 2[\frac{1}{2}r^2 \sin 60^{\circ}] + 4[(\frac{\pi}{6})r^2 - \frac{\sqrt{3}}{4}r^2]$

$A = \frac{\sqrt{3}}{2}r^2 + 2(\frac{\pi}{3})r^2 - \sqrt{3}r^2 = \frac{2}{3}\pi r^2 - \frac{1}{2}\sqrt{3}r^2$

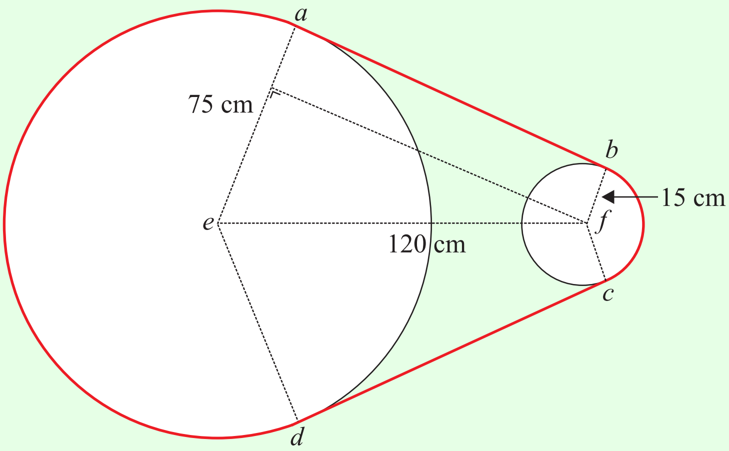
$\therefore A = 2r^2(\frac{\pi}{3} - \frac{\sqrt{3}}{4})$

Sector: $A = \frac{1}{2}r^2\theta$ 7

Triangle: $A = \frac{1}{2}ab\sin C$ 5

2002

4 (c) A chain passes around two circular wheels as shown. One wheel has a radius 75 cm and the other has radius 15 cm. The centres, *e* and *f*, of the wheels are 120 cm apart. The chain consists of the common tangent [*ab*], the minor arc *bc*, the common tangent [*cd*] and the major arc *da*.



(i) Find the measure of $\angle aef$.

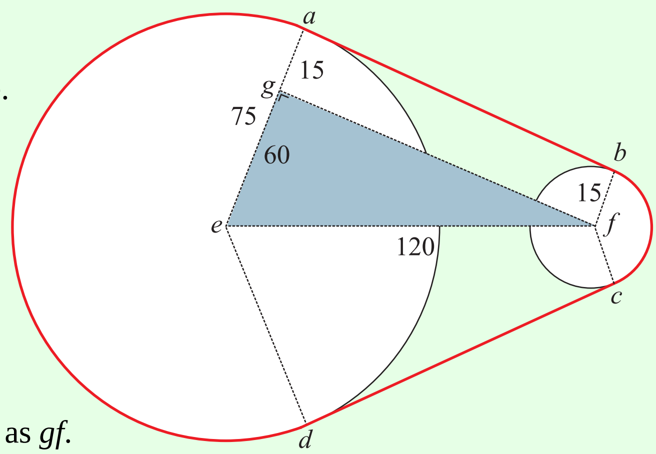
(ii) Find $|ab|$ in surd form.

(iii) Find the length of the chain, giving your answer in the form $k\pi + l\sqrt{3}$ where $k, l \in \mathbf{Z}$.

Solution

4 (c) (i)

Consider the blue right-angled triangle.



$$\cos\theta = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \text{.......} \; (1)$$

$$\cos\angle aef = \frac{60}{120} = \frac{1}{2} \Rightarrow \angle aef = 60^\circ$$

4 (c) (ii)

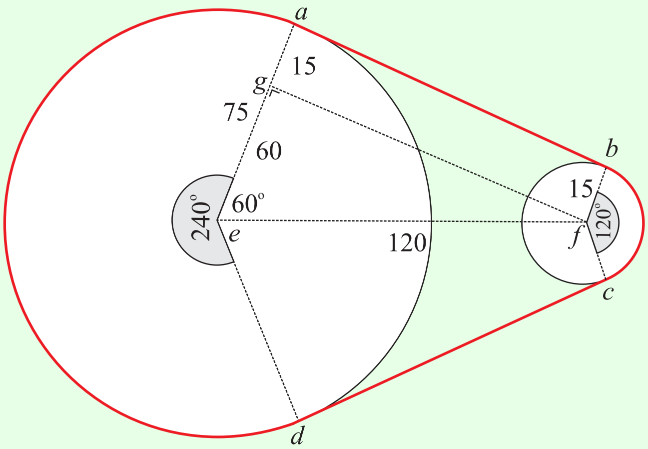
ab is parallel to and is the same length as *gf*.

$$60^2 + |gf|^2 = 120^2 \Rightarrow |gf|^2 = 10{,}800$$

$$\Rightarrow |gf| = \sqrt{10{,}800} = \sqrt{3600 \times 3} = 60\sqrt{3} = |ab|$$

4 (c) (iii)

Length of chain $L = |ab| + |cd| +$ Small arc bc + Large arc ad



$\therefore L = 60\sqrt{3} + 60\sqrt{3} + 15(\frac{2\pi}{3}) + 75(\frac{4\pi}{3})$

$\Rightarrow L = 120\sqrt{3} + 10\pi + 100\pi = 120\sqrt{3} + 110\pi$

Arc length s

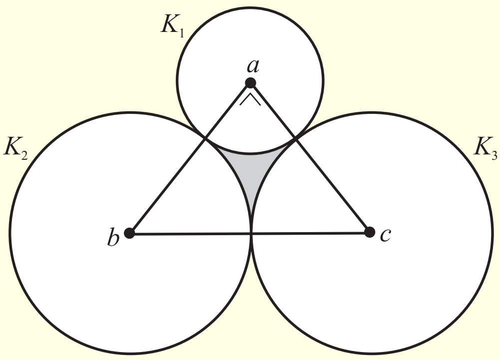
$s = r\theta$ 6

2004

4 (c) a, b and c are the centres of the circles K_1, K_2 and K_3 repectively. The three circles touch externally and $ab \perp ac$. K_2 and K_3 each have radius $2\sqrt{2}$ cm.

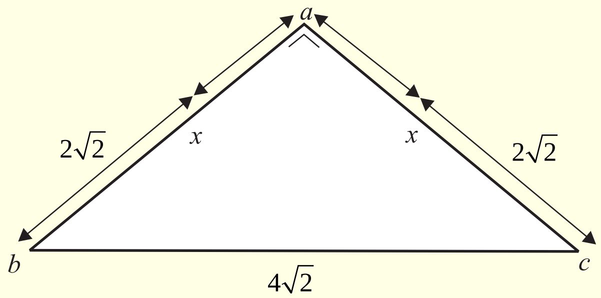
(i) Find, in surd form, the length of the radius of K_1.

(ii) Find the area of the shaded region in terms of π.



SOLUTION

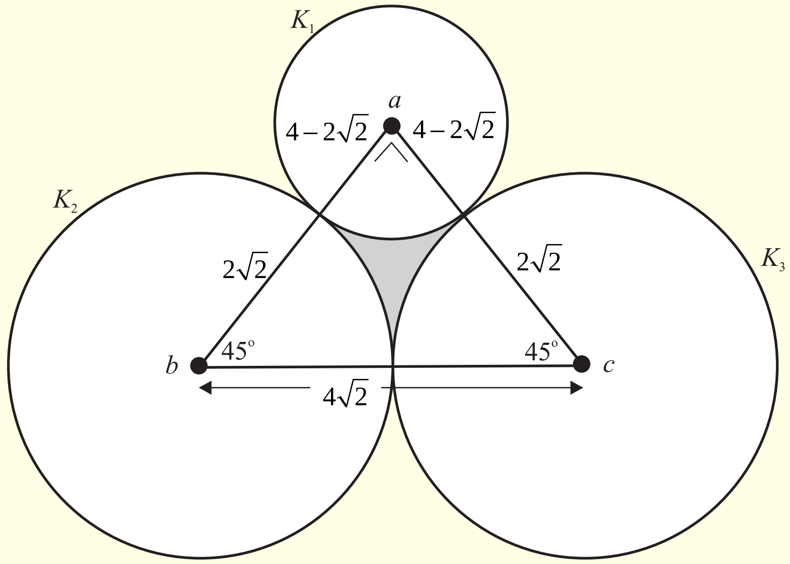
4 (c) (i)



Using Pythagoras on Δabc: $(4\sqrt{2})^2 = x^2 + x^2 \Rightarrow 32 = 2x^2 \Rightarrow x^2 = 16 \Rightarrow x = 4$

Therefore, radius of K_1 is $4 - 2\sqrt{2}$.

4 (c) (ii)



Area of shaded region = Area of Δabc – (Area of the three sectors)

Area of a triangle: $A = \frac{1}{2} \times \text{Base} \times \text{Height}$

$A = \frac{1}{2} r^2 \theta$ 7

$\therefore A = \frac{1}{2}(4)(4) - \frac{1}{2}(2\sqrt{2})^2(\frac{\pi}{4}) - \frac{1}{2}(2\sqrt{2})^2(\frac{\pi}{4}) - \frac{1}{2}(4 - 2\sqrt{2})^2(\frac{\pi}{2})$

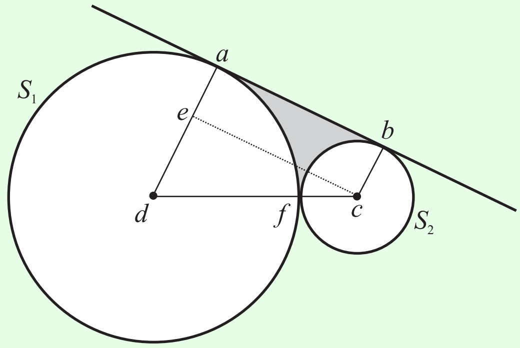
$\therefore A = \frac{1}{2}(4)(4) - \frac{1}{2}(2\sqrt{2})^2(\frac{\pi}{4}) - \frac{1}{2}(2\sqrt{2})^2(\frac{\pi}{4}) - \frac{1}{2}(4 - 2\sqrt{2})^2(\frac{\pi}{2})$

$\Rightarrow A = 8 - \pi - \pi - (\frac{\pi}{4})(16 - 16\sqrt{2} + 8) \Rightarrow A = 8 - 2\pi - 6\pi + 4\sqrt{2}\pi$

$\Rightarrow A = 8 - 8\pi + 4\sqrt{2}\pi$

2005

4 (c) S_1 is a circle of radius 9 cm and S_2 is a circle of radius 3 cm. S_1 and S_2 touch externally at *f*. A common tangent touches S_1 at point *a* and S_2 at *b*.

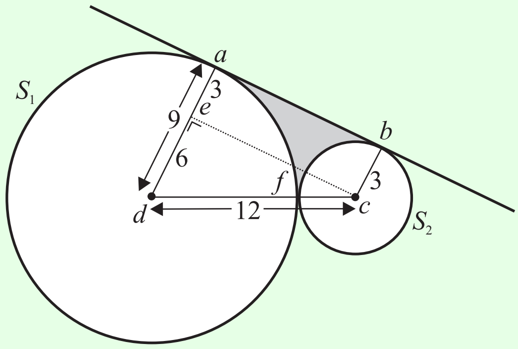


(i) Find the area of the quadrilateral *abcd*. Give your answer in surd form.

(ii) Find the area of the shaded region, which is bounded by [*ab*] and the monor arcs *af* and *bf*.

Solution

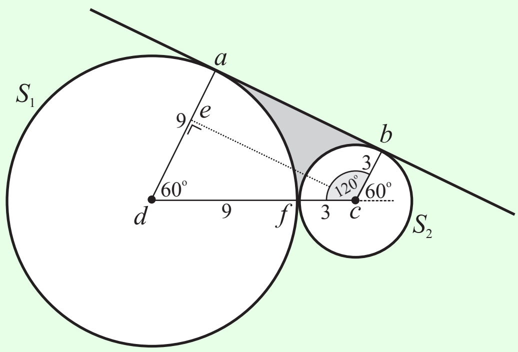
4 (c) (i)



Using Pythagoras on Δdec: $12^2 = 6^2 + |ec|^2 \Rightarrow |ec| = \sqrt{144-36} = \sqrt{108} = 6\sqrt{3}$

Area of *abcd* = Area of Δdec + Area of rectangle *abce*

$= \frac{1}{2}(6)(6\sqrt{3}) + 3(6\sqrt{3}) = 18\sqrt{3} + 18\sqrt{3} = 36\sqrt{3}$



4 (c) (ii)

Before you do this, calculate some angles.

$\cos\angle edc = \frac{12}{6} = \frac{1}{2} \Rightarrow \angle edc = 60^\circ = \frac{\pi}{3}$

As $ad \parallel bc \Rightarrow \angle bcf = 180^\circ - 60^\circ = 120^\circ = \frac{2\pi}{3}$

Shaded area = Area of *abcd* – Area of sector *adf* – Area of sector *bcf*

$A = \frac{1}{2}r^2\theta$ **7**

Shaded area $= 36\sqrt{3} - \frac{1}{2}(9)^2(\frac{\pi}{3}) - \frac{1}{2}(3)^2(\frac{2\pi}{3}) = 36\sqrt{3} - \frac{27}{2}\pi - 3\pi$

$= 36\sqrt{3} - \frac{33}{2}\pi$