TRIGONOMETRY (Q 4 & 5, PAPER 2)

2009

4 (a) Show that $(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2=2$.

(b) The lengths of the sides of a triangle are 21, 17 and 10.
The smallest angle in the triangle is A.

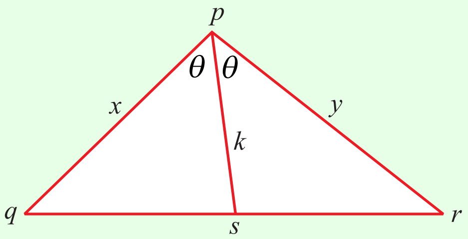
(i) Show that $\cos A=\frac{15}{17}$.

(ii) Without evaluating A, find $\tan\frac{A}{2}$.

(c) The bisector of $\angle qpr$ meets $[qr]$ at s.

$|\angle qpr|=2\theta$, $|pq|=x$,

$|pr|=y$ and $|ps|=k$.



(i) Find the area of the triangle pqs in terms of x, k and θ.

(ii) Show that $k=\dfrac{2xy\cos\theta}{x+y}$.

SOLUTION

4 (a)

LHS

$$(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2$$
$$=\cos^2\theta+2\cos\theta\sin\theta+\sin^2\theta+\cos^2\theta-2\cos\theta\sin\theta+\sin^2\theta$$
$$=\cos^2\theta+\sin^2\theta+\cos^2\theta+\sin^2\theta \qquad \boxed{\cos^2 A+\sin^2 A=1}$$
$$=1+1$$
$$=2$$

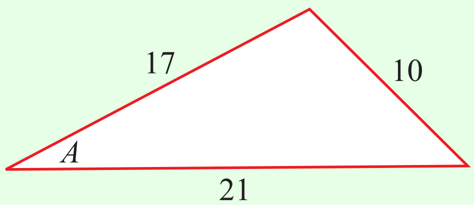
RHS

2

4 (b) (i)

$$\boxed{a^2=b^2+c^2-2bc\cos A}$$

$$10^2=17^2+21^2-2(17)(21)\cos A$$
$$714\cos A=17^2+21^2-10^2$$
$$714\cos A=630$$
$$\cos A=\frac{630}{714}=\frac{15}{17}$$



4 (b) (ii)

$\cos A = \frac{15}{17}$

$\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

$15^2 + y^2 = 17^2$

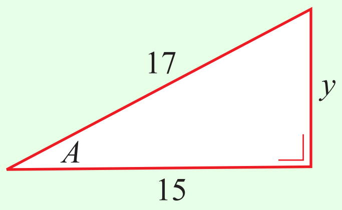
$x^2 + y^2 = r^2$

$y^2 = 17^2 - 15^2 = 64$

$y = \sqrt{64} = 8$

$\therefore \tan A = \frac{8}{15}$

$\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}$



Let $B = \frac{A}{2} \Rightarrow A = 2B$

$\therefore \tan 2B = \frac{8}{15}$

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$\frac{2\tan B}{1-\tan^2 B} = \frac{8}{15}$

$30\tan B = 8 - 8\tan^2 B$

$4\tan^2 B + 15\tan B - 4 = 0$

$(4\tan B - 1)(\tan B + 4) = 0$

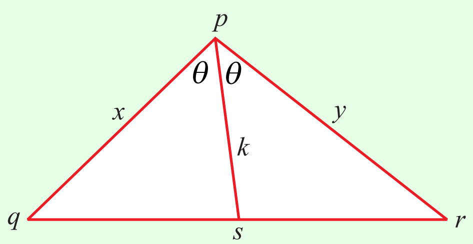
$\therefore \tan B = \frac{1}{4}$, ~~$\tan B = -4$~~

$\therefore \tan \frac{A}{2} = \frac{1}{4}$

4 (c) (i)

Triangle *pqs*: $A_1 = \frac{1}{2}kx\sin\theta$

$A = \frac{1}{2}ab\sin C$



4 (c) (ii)

Triangle *pqr* (A) = Triangle *pqs* (A_1) + Triangle *prs* (A_2)

Triangle *pqr*: $A = \frac{1}{2}xy\sin 2\theta$

Triangle *pqs*: $A_1 = \frac{1}{2}kx\sin\theta$

Triangle *prs*: $A_2 = \frac{1}{2}ky\sin\theta$

$\therefore \frac{1}{2}xy\sin 2\theta = \frac{1}{2}kx\sin\theta + \frac{1}{2}ky\sin\theta$

$2xy\sin\theta\cos\theta = kx\sin\theta + ky\sin\theta$

$2xy\cos\theta = k(x+y)$

$\frac{2xy\cos\theta}{x+y} = k$

5 (a) Find all the solutions of the equation $\cos^2 x - \cos x = 0$, where $0^\circ \le x \le 180^\circ$.

(b) The function $f: x \to \sin^{-1} x$ is defined for $-1 \le x \le 1$.

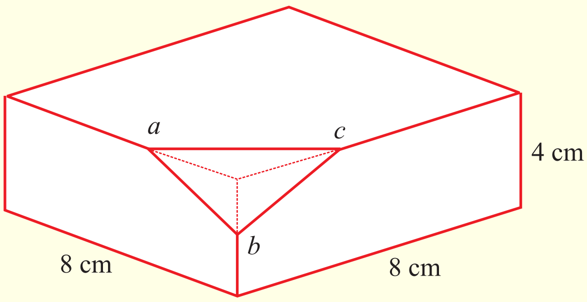
(i) Copy and complete the table of values of f below.

x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1
$f(x)$			$-\frac{\pi}{6}$				

(ii) Draw the graph of $y = f(x)$ on graph paper, noting that $\frac{\sqrt{3}}{2} \approx 0.87$.
Scale the y-axis in terms of π.

(iii) State, with reason, whether each of the following statements is true.
A: "If $\sin x_1 = \sin x_2$, then $x_1 = x_2$."
B: "If $\sin^{-1} x_1 = \sin^{-1} x_2$, then $x_1 = x_2$."

(c) A rectangular block of cheese measures $8 \text{ cm} \times 8 \text{ cm} \times 4 \text{ cm}$.
One corner is cut away from the block, such a way that three of the edges are cut through their midpoints a, b and c. Find the area of the triangular face abc created by the cut.



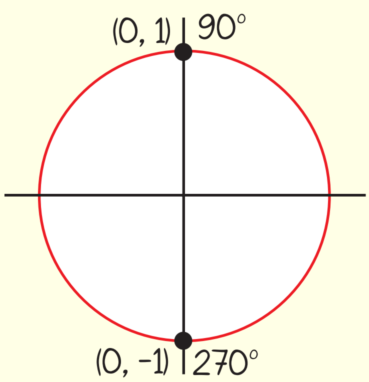
SOLUTION

5 (a)

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

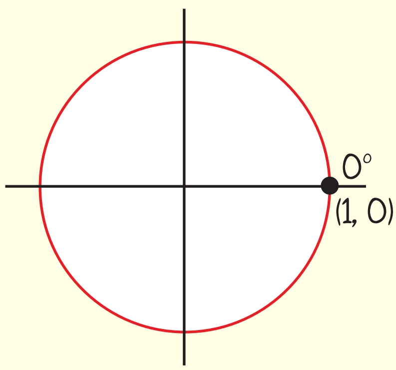
$\cos x = 0$



$x = 90^\circ$

$\cos x - 1 = 0$

$\cos x = 1$



$x = 0^\circ$

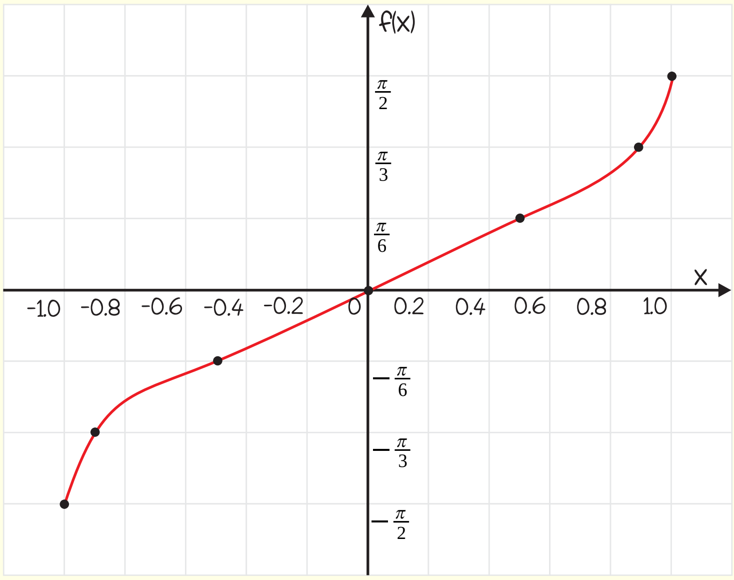
ANS: $x = 0^\circ, 90^\circ$

5 (b) (i)

x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1
$f(x)$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$

[Use your calculator in radian mode.]

5 (b) (ii)



5 (b) (iii)

Statement A is false.

$\sin 150^\circ = \sin 30^\circ = \frac{1}{2}$ even though $150^\circ \neq \sin 30^\circ$. A horizontal line can cut the graph of $y = \sin x$ more than once.

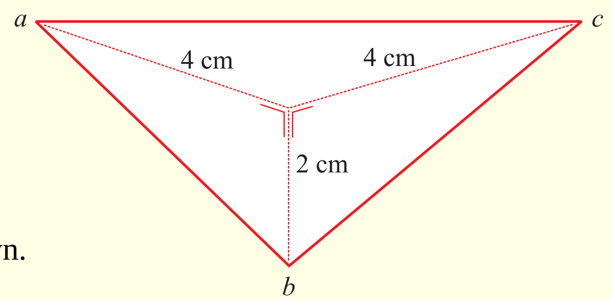
Statement B is true. As can be seen for the graph above of $y = \sin^{-1} x$ a horizontal line cannot cut the graph more than once.

5 (c)

$|ab|^2 = 2^2 + 4^2 = 20 \Rightarrow |ab| = \sqrt{20} = 2\sqrt{5}$

$|bc|^2 = 2^2 + 4^2 = 20 \Rightarrow |bc| = \sqrt{20} = 2\sqrt{5}$

$|ac|^2 = 4^2 + 4^2 = 32 \Rightarrow |ac| = \sqrt{32} = 4\sqrt{2}$



The triangle is isosceles. Turn it upside down.

$h^2 + (2\sqrt{2})^2 = (2\sqrt{5})^2$

$h^2 + 8 = 20$

$h^2 = 12$

$h = \sqrt{12} = 2\sqrt{3}$

Area $= \frac{1}{2}bh = \frac{1}{2} \times 4\sqrt{2} \times 2\sqrt{3} = 4\sqrt{6}$ cm^2