TRIGONOMETRY (Q 4 & 5, PAPER 2)

2007

4 (a) Show that $(\cos A+\sin A)^2=1+\sin 2A$.

(b) Find all the solutions of the equation

$6\cos^2 x+\sin x-5=0$, where $0^\circ \le x \le 360^\circ$.

Give the solutions correct to the nearest degree.

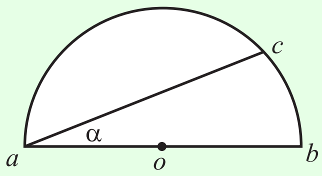
(c) $[ab]$ is the diameter of a semicircle of centre o and radius-length r.

$[ac]$ is a chord such that $|\angle cab|=\alpha$, where α is in radian measure.

(i) Find $|ac|$ in terms of r and α.

(ii) $[ac]$ bisects the area of the semicircular region.

Show that $2\alpha+\sin 2\alpha=\frac{\pi}{2}$.



SOLUTION

4 (a)

LHS	RHS
$(\cos A+\sin A)^2$	$1+\sin 2A$
$=\cos^2 A+2\cos A\sin A+\sin^2 A$	
$=(\cos^2 A+\sin^2 A)+2\cos A\sin A$	
$=1+\sin 2A$	

$\cos^2 A+\sin^2 A=1$ **8**

$\sin 2A=2\sin A\cos A$ **13**

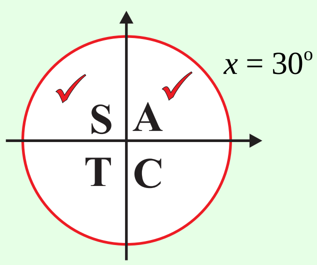
4 (b)

$6\cos^2 x+\sin x-5=0 \Rightarrow 6(1-\sin^2 x)+\sin x-5=0$

$\Rightarrow 6-6\sin^2 x+\sin x-5=0 \Rightarrow 6\sin^2 x-\sin x-1=0$

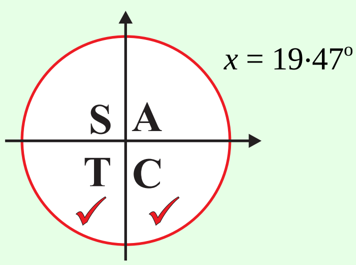
$\Rightarrow (3\sin x+1)(2\sin x-1)=0 \Rightarrow \sin x=-\frac{1}{3},\frac{1}{2}$

$\sin x=\frac{1}{2}$



$x = 30^\circ$ (First quadrant)
$= 150^\circ$ (Second quadrant)

$\sin x=-\frac{1}{3}$



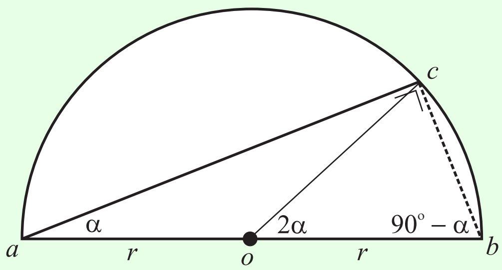
$x = 199^\circ$ (Third quadrant)
$= 341^\circ$ (Fourth quadrant)

$x = 30^\circ, 150^\circ, 199^\circ, 341^\circ$

4 (c) (i)

$\angle acb$ is right-angled. Consider triangle *acb*.

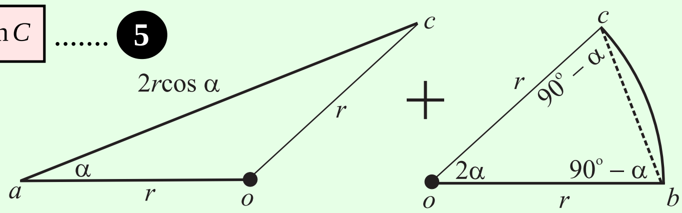
$\cos A = \frac{x}{r} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ 1



$\cos\alpha = \frac{|ac|}{2r} \Rightarrow |ac| = 2r\cos\alpha$

4 (c) (ii)

$A = \frac{1}{2}ab\sin C$ 5



$A = \frac{1}{2}r^2\theta$ 7

Area of *abc* = $\frac{1}{4}\times\pi r^2$

This area is made up of the triangle *aoc* and the sector *obc*.

$\therefore \frac{1}{2}(r)(2r\cos\alpha)\sin\alpha + \frac{1}{2}r^2(2\alpha) = \frac{1}{4}\times\pi r^2$

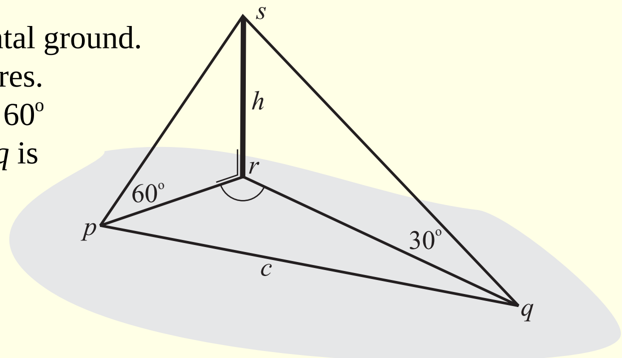
$\Rightarrow 2\cos\alpha\sin\alpha + 2\alpha = \frac{\pi}{2}$

$\Rightarrow 2\alpha + \sin 2\alpha = \frac{\pi}{2}$

5 (a) Evaluate $\lim\limits_{x\to 0}\frac{\sin 2x}{\sin 3x}$.

(b) Using the formula $\cos(A+B) = \cos A\cos B - \sin A\sin B$, derive a formula for $\cos(A-B)$ and hence prove that $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

(c) *p*, *q* and *r* are three points on horizontal ground.
[*sr*] is a vertical pole of height *h* metres.
The angle of elevation of *s* from *p* is 60°
and the angle of elevation of *s* from *q* is 30°.
| *pq*| = *c* metres.
Given that $13c^2 = 13h^2$, find $|\angle prq|$.



SOLUTION

5 (a)

$$\lim_{x\to 0}\frac{\sin 2x}{\sin 3x} = \lim_{x\to 0}\left(\frac{\sin 2x}{2x}\times\frac{3x}{\sin 3x}\times\frac{2x}{3x}\right) = \frac{2}{3}$$

$\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ 20

OR

$\lim\limits_{\theta\to 0}\frac{\theta}{\sin\theta} = 1$ 20

5 (b)

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

Replace B by $-B$.

$\Rightarrow \cos(A-B) = \cos A \cos(-B) - \sin A \sin(-B)$

$\cos(-A) = \cos A$	$\sin(-A) = -\sin A$

$\Rightarrow \cos(A-B) = \cos A \cos B + \sin A \sin B$

Replace A by $(90^\circ - A)$.

$\Rightarrow \cos((90^\circ - A) - B) = \cos(90^\circ - A)\cos B + \sin(90^\circ - A)\sin B$

$\Rightarrow \cos(90^\circ - (A+B)) = \cos(90^\circ - A)\cos B + \sin(90^\circ - A)\sin B$

$\Rightarrow \sin(A+B) = \sin A \cos B + \cos A \sin B$

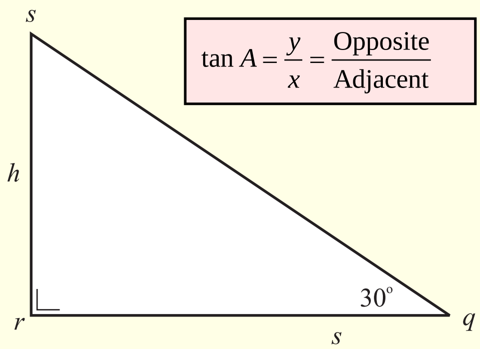
5 (c)

STEPS
1. Identify all right-angled triangles and non right-angled triangles and mark all angles and sides and label all vertices.
2. Separate out the triangles.

Consider the right-angled triangle srq:

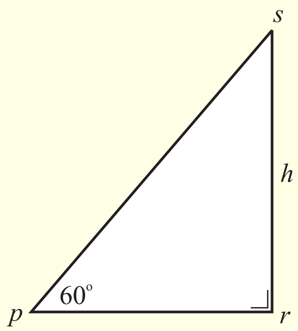
$\tan 30^\circ = \frac{h}{|rq|} \Rightarrow |rq| = \frac{h}{\tan 30^\circ} = \frac{h}{\frac{1}{\sqrt{3}}} = \sqrt{3}h$

$\tan A = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}$ 3



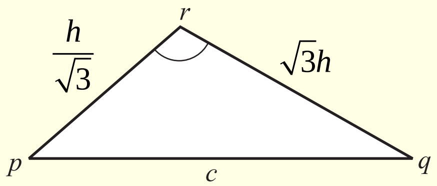
Consider the right-angled triangle srp:

$\tan 60^\circ = \frac{h}{|rp|} \Rightarrow |rp| = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}}$



Consider triangle rpq:

$a^2 = b^2 + c^2 - 2bc\cos A$ 19



$c^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + (\sqrt{3}h)^2 - 2\left(\frac{h}{\sqrt{3}}\right)(\sqrt{3}h)\cos|\angle prq|$

$\Rightarrow c^2 = \frac{1}{3}h^2 + 3h^2 - 2h^2\cos|\angle prq| \Rightarrow 3c^2 = h^2 + 9h^2 - 6h^2\cos|\angle prq|$

Using $3c^2 = 13h^2 \Rightarrow 13h^2 = 10h^2 - 6h^2\cos|\angle prq| \Rightarrow 3h^2 = -6h^2\cos|\angle prq|$

$\Rightarrow -\frac{1}{2} = \cos|\angle prq| \Rightarrow \cos|\angle prq| = 120^\circ$