TRIGONOMETRY (Q 4 & 5, PAPER 2)

2006

4 (a) Write down the values of A for which $\cos A = \frac{1}{2}$, where $0^\circ \le A \le 360^\circ$.

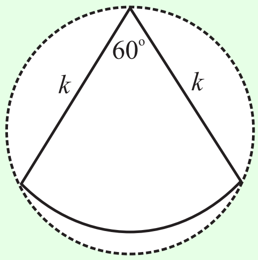
4 (b) (i) Express $\sin(3x + 60^\circ) - \sin x$ as a product of sine and cosine.

(ii) Find all the solutions of the equation $\sin(3x + 60^\circ) - \sin x = 0$, where $0^\circ \le A \le 360^\circ$.

4 (c) The diagram shows a sector (solid line) circumscribed by a circle (dashed line).

(i) Find the radius of the circle in terms of k.

(ii) Show that the circle encloses an area which is double that of the sector.



SOLUTION

4 (a)

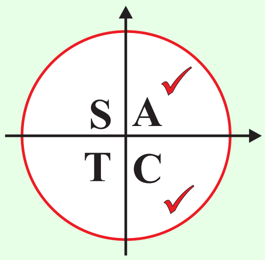
$\cos A = \frac{1}{2} \Rightarrow A = 60^\circ$

Cosine is positive in the first and fourth quadrants.

$A = 60^\circ$ (First quadrant)

$A = 360^\circ - 60^\circ = 300^\circ$ (Fourth quadrant)

Ans: $60^\circ, 300^\circ$



4 (b) (i)

$$\sin(3x + 60^\circ) - \sin x = 2\cos\left(\frac{4x + 60^\circ}{2}\right)\sin\left(\frac{2x + 60^\circ}{2}\right)$$

$$= 2\cos(2x + 30^\circ)\sin(x + 30^\circ)$$

SUMS → PRODUCTS

$\sin A + \sin B = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$

$\sin A - \sin B = 2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$

$\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$

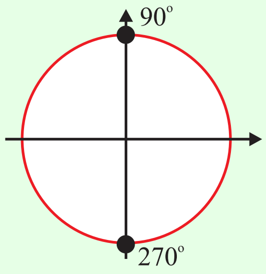
$\cos A - \cos B = -2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$

4 (b) (ii)

$\sin(3x+60^\circ)-\sin x=0 \Rightarrow 2\cos(2x+30^\circ)\sin(x+30^\circ)=0$

$\Rightarrow \cos(2x+30^\circ)\sin(x+30^\circ)=0$

$\Rightarrow \cos(2x+30^\circ)=0$



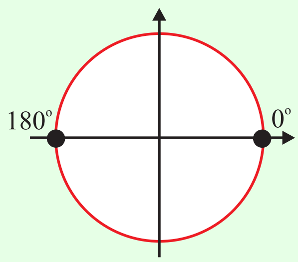
$2x+30^\circ = 90^\circ,\ 450^\circ$

$= 270^\circ,\ 630^\circ$

$x = 30^\circ,\ 210^\circ$

$= 120^\circ,\ 300^\circ$

$\Rightarrow \sin(x+30^\circ)=0$



$x+30^\circ = 0^\circ,\ 360^\circ$

$= 180^\circ$

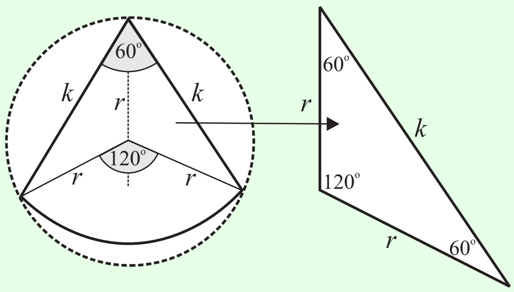
$x = 330^\circ$

$= 150^\circ$

Ans: $30^\circ, 120^\circ, 150^\circ, 210^\circ, 300^\circ, 330^\circ$

4 (c) (i)

From the Junior Cert., you might remember that the angle at the centre of a circle is twice the angle on the circle standing on the same arc.



$a^2=b^2+c^2-2bc\cos A$ **19**

Using the Cosine rule: $k^2=r^2+r^2-2(r)(r)\cos120^\circ \Rightarrow k^2=2r^2-2r^2\cos120^\circ$

$\Rightarrow k^2=2r^2(1-\cos120^\circ) \Rightarrow k^2=2r^2(1-\cos120^\circ)$

$\Rightarrow k^2=2r^2(1+\cos60^\circ) \Rightarrow k^2=2r^2(1+\tfrac{1}{2}) \Rightarrow k^2=2r^2(\tfrac{3}{2})$

$\Rightarrow k^2=3r^2 \Rightarrow r=\frac{k}{\sqrt{3}}$

4 (c) (ii)

Area of circle: $A_1=\pi r^2=\pi\left(\frac{k^2}{3}\right)=\tfrac{1}{3}\pi k^2$

Area of sector: $A_2=\tfrac{1}{2}k^2(\tfrac{\pi}{3})=\tfrac{1}{6}\pi k^2$ $A=\tfrac{1}{2}r^2\theta$ **7**

$\therefore A_1=2A_2$

5 (a) (i) Copy and complete the table below for $f:\ x \to \tan^{-1} x,$ giving the values for $f(x)$ in terms of π.

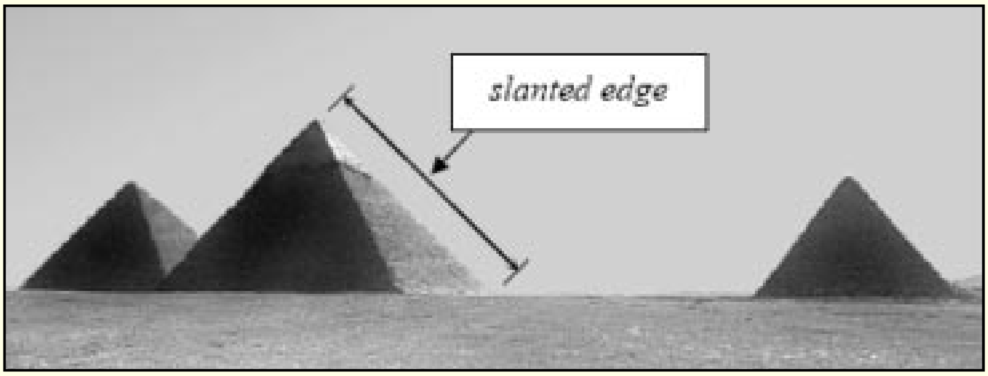
x	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$f(x)$						$\frac{\pi}{4}$	

(ii) Draw a graph of $y = f(x)$ in the domain $-2 \le x \le 2,$ scaling the y-axis in terms of π.

(iii) Draw the two horizontal asymptotes of the graph.

(iv) For some values of $k \in \mathbf{R},$ but not all values, $\tan^{-1}(\tan k) = k.$ State the range of values of k for which $\tan^{-1}(\tan k) = k.$ Show, by means of an example, what happens outside the range.

(b) The great pyramid at Giza in Eqypt has a square base and four triangular faces. The base of the pyramid is of side 230 metres and the pyramid is 146 metres high. The top of the pyramid is directly above the centre of the base.



(i) Calculate the length of one of the slanted edges, correct to the nearest metre.

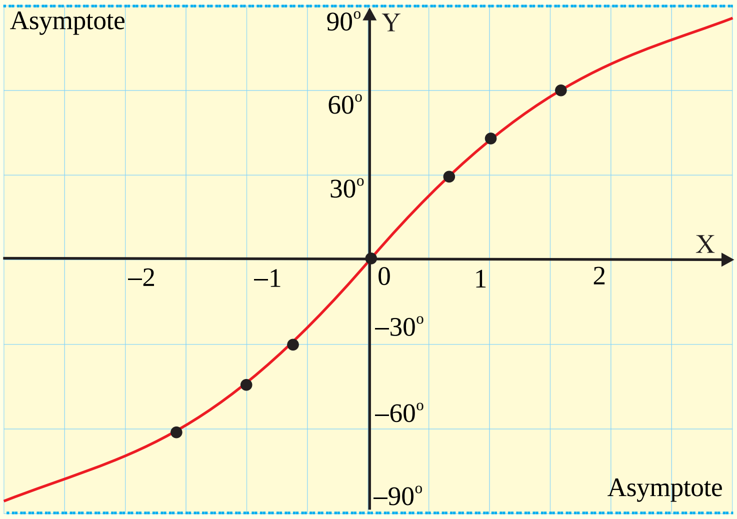
(ii) Calculate, correct to two significant figures, the total area of the four triangular faces of the pyramid (assuming they are smooth flat surfaces).

Solution

5 (a) (i)

x	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$f(x)$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$

5 (a) (ii)
5 (a) (iii)



5 (a) (iv)

You can see from the graph the range of values which apply to $\tan^{-1} x$: $-\frac{\pi}{2} \le k \le \frac{\pi}{2}$.

Choose a value outside this range like 120°.

Ex. $\tan^{-1}(\tan 120^\circ) = -60^\circ$ [Use your calculator to show this.]

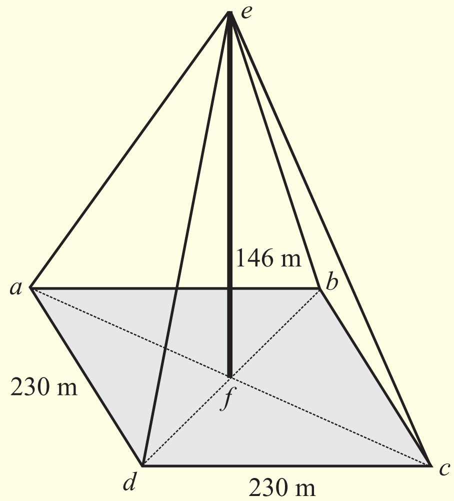
You can see when you go outside the range from -90° to 90°, that $\tan^{-1}(\tan k) = k$ does not apply.

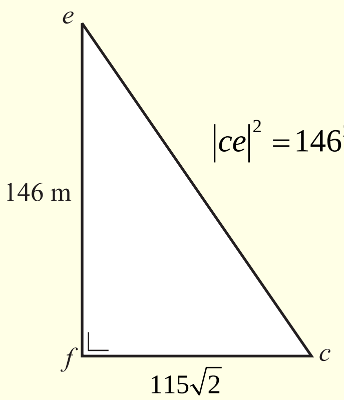
5 (b) (i)

Using Pythagoras calculate the length of the diagonal of the square base.

$|bd|^2 = 230^2 + 230^2 \Rightarrow |bd| = 230\sqrt{2}$

Pick out Δefc.



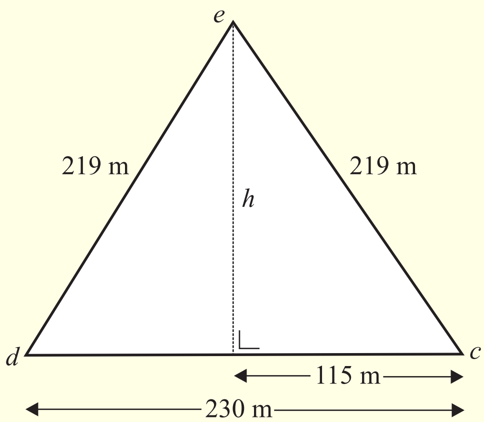


$|ce|^2 = 146^2 + (115\sqrt{2})^2 \Rightarrow |ce| = 219$ m

5 (b) (ii)

Pick out Δedc. Using Pythagoras, calculate the height h of the triangular face.

$\therefore h^2 + 115^2 = 219^2 \Rightarrow h = 186$ m



Area of a triangle: $A = \frac{1}{2} \times \text{Base} \times \text{Height}$

Area of four triangular faces:

$A = 4 \times \frac{1}{2} \times 230 \times 186 = 85{,}560 \text{ m}^2 \approx 86{,}000 \text{ m}^2$ [Correct to 2 significant figures.]