(© Tony Kelly & Kieran Mills)

TRIGONOMETRY (Q 4 & 5, PAPER 2)

2003

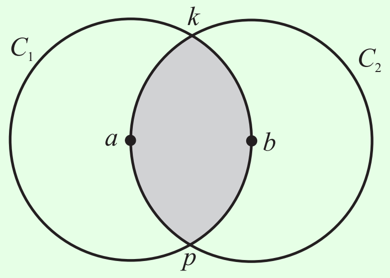
4 (a) The circumference of a circle is 30π cm. The area of a sector of the circle is 75 cm^2. Find, in radians, the angle in this sector.

4 (b) Find all the solutions of the equation $\sin 2x + \sin x = 0$ in the domain $0^\circ \le x \le 360^\circ$.

4 (c) C_1 is a circle with centre a and radius r. C_2 is a circle with centre b and radius r. C_1 and C_2 intersect at k and p. $a \in C_2$. $b \in C_1$.

(i) Find, in radians, the measure of angle kap.

(ii) Calculate the area of the shaded region. Give your answer in terms of r and π.

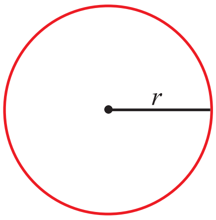


SOLUTION

4 (a)

Use page 6 & 7 of the tables:

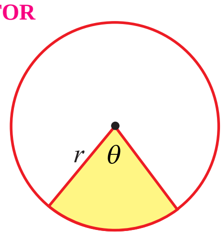
CIRCLE



Length $= 2\pi r$

Area $= \pi r^2$

SECTOR



Length $= r\theta$ (θ in radians)

Area $= \frac{1}{2}r^2\theta$ (θ in radians)

$2\pi r = 30\pi \Rightarrow r = 15$ cm

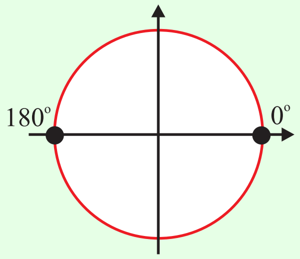
$$A = \tfrac{1}{2}(15)^2\theta = 75 \Rightarrow \theta = \frac{2\times 75}{225} = \tfrac{2}{3} \text{ rad}$$

4 (b)

$\sin 2x + \sin x = 0 \Rightarrow 2\sin x \cos x + \sin x = 0$

$\Rightarrow \sin x(2\cos x + 1) = 0$

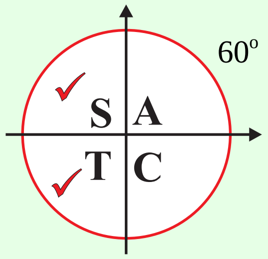
$\Rightarrow \sin x = 0$



$x = 0^\circ, 360^\circ$

$= 180^\circ$

$\Rightarrow \cos x = -\frac{1}{2}$



$x = 120^\circ$ [Second quadrant]

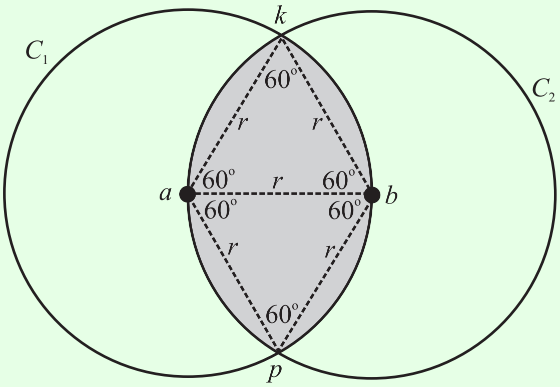
$= 240^\circ$ [Third quadrant]

Ans: $0^\circ, 120^\circ, 180^\circ, 240^\circ, 360^\circ$

4 (c) (i)

Δakb and Δpab are equilateral as all sides are equal to the radius r. Therefore, all angles are 60°.

$\angle kap = 120^\circ = \frac{2\pi}{3}$



4 (c) (ii)

Area of the shaded region is the area of the 2 blue triangles plus the 4 grey segments.
To find the area of a grey segment:
Area of sector kab – Area of Δkab

$= \frac{1}{2}r^2(\frac{\pi}{3}) - \frac{1}{2}(r)(r)\sin 60^\circ$

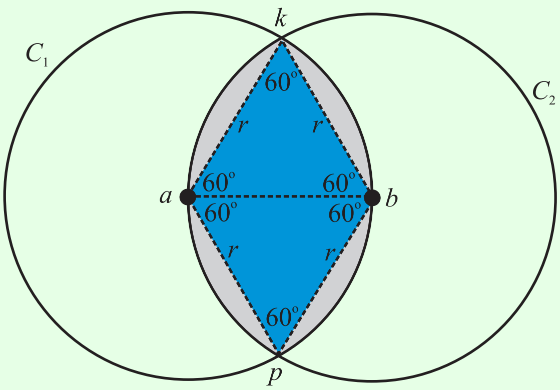
$= (\frac{\pi}{6})r^2 - \frac{\sqrt{3}}{4}r^2$

Area of shaded region:

$A = 2[\frac{1}{2}r^2 \sin 60^\circ] + 4[(\frac{\pi}{6})r^2 - \frac{\sqrt{3}}{4}r^2]$

$A = \frac{\sqrt{3}}{2}r^2 + 2(\frac{\pi}{3})r^2 - \sqrt{3}r^2 = \frac{2}{3}\pi r^2 - \frac{1}{2}\sqrt{3}r^2$

$\therefore A = 2r^2(\frac{\pi}{3} - \frac{\sqrt{3}}{4})$



Sector: $A = \frac{1}{2}r^2\theta$ **7**

Triangle: $A = \frac{1}{2}ab\sin C$ **5**

(© Tony Kelly & Kieran Mills)

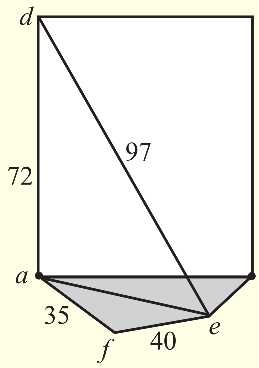
5 (a) Find the value of $\sin 15^\circ$ in surd form.

5 (b) a, f and e are points on horizontal ground. d is a point on a vertical wall directly above a.

$|ad| = 72$ m, $|de| = 97$ m, $|af| = 35$ m and $|fe| = 40$ m.

(i) Calculate $|ae|$.

(ii) Hence, calculate $|\angle afe|$.



5 (c) (i) Using the identity $\cos(A-B) = \cos A\cos B + \sin A\sin B$, or otherwise, prove:

$\sin(A+B) = \sin A\cos B + \cos A\sin B$.

(ii) Prove: $\sin(A+B)\sin(A-B) = (\sin A + \sin B)(\sin A - \sin B)$.

Solution

5 (a)

$\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

A	0	π	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	[Radians]
A	0°	180°	90°	60°	45°	30°	[Degrees]
$\cos A$	1	-1	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	
$\sin A$	0	0	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	
$\tan A$	0	0	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	

$\sin(A-B) = \sin A\cos B - \cos A\sin B$ **10**

5 (b) (i)

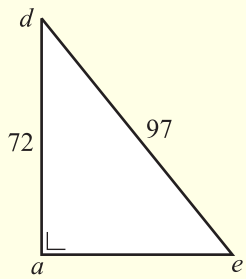
Steps

1. Identify all right-angled triangles and non right-angled triangles and mark all angles and sides and label all vertices.
2. Separate out the triangles.

Pick out the right-angled triangle Δdae.

Using Pythagoras $72^2 + |ae|^2 = 97^2 \Rightarrow |ae|^2 = 9409 - 5184 = 4225$

$\Rightarrow |ae| = \sqrt{4225} = 65$



5 (b) (ii)

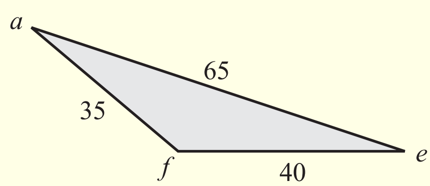
Pick out Δafe. Use the Cosine rule.

$a^2 = b^2 + c^2 - 2bc\cos A$ **19**

$65^2 = 35^2 + 40^2 - 2(35)(40)\cos|\angle afe|$

$1400 = -2800\cos|\angle afe| \Rightarrow \cos|\angle afe| = -\frac{1}{2}$

$\Rightarrow |\angle afe| = 120^\circ$

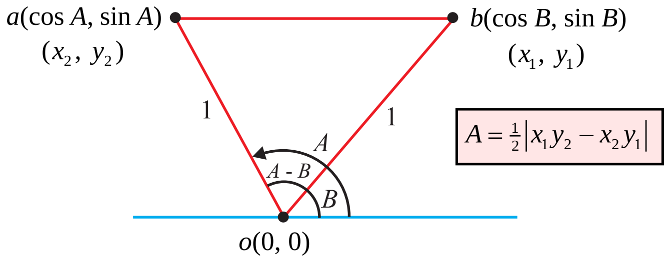


(© Tony Kelly & Kieran Mills)

5 (c) (i)

The method for proving compound angle formulae involves using the diagram below to prove the formula for $\sin(A-B)$ and then derive the other formulae from this one.

PROOF: $\sin(A-B)=\sin A\cos B-\cos A\sin B$



Area $\Delta oba=\frac{1}{2}\times1\times1\times\sin(A-B)=\frac{1}{2}(\cos B\sin A-\sin B\cos A)$

$\Rightarrow \sin(A-B)=\sin A\cos B-\cos A\sin B$ .... Formula **10**

Replacing B by $-B$ in Formula **10**

$\Rightarrow \sin(A+B)=\sin A\cos(-B)-\cos A\sin(-B)$

$\Rightarrow \sin(A+B)=\sin A\cos B+\cos A\sin B$ .... Formula **9**

Using another method as directed by the question:

You are told that $\cos(A-B)=\cos A\cos B+\sin A\sin B$.

Replace A by $90^\circ-A$: $\therefore \cos((90^\circ-A)-B)=\cos(90^\circ-A)\cos B+\sin(90^\circ-A)\sin B$

$\Rightarrow \cos(90^\circ-(A+B))=\sin A\cos B+\cos A\sin B$

$\Rightarrow \sin(A+B)=\sin A\cos B+\cos A\sin B$

5 (c) (ii)

This is a trig identity.

STEPS

1. You have to prove that the left-hand side (*LHS*) equals the right-hand side (*RHS*).
2. Change everything to sine and cosine.
3. Simplify each side using page 9 of the tables and good algebra.
4. For half angles, $\frac{\theta}{2}$: Let $\frac{\theta}{2}=A\Rightarrow\theta=2A$.

LHS

$\sin(A+B)\sin(A-B)$

$=(\sin A\cos B+\cos A\sin B)(\sin A\cos B-\cos A\sin B)$

$=\sin^2 A\cos^2 B+\cos A\sin B\sin A\cos B-\cos A\sin B\sin A\cos B-\cos^2 A\sin^2 B$

$=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B$

$=\sin^2 A(1-\sin^2 B)-(1-\sin^2 A)\sin^2 B$

$=\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B-\sin^2 A\sin^2 B$

$=\sin^2 A-\sin^2 B=(\sin A+\sin B)(\sin A-\sin B)$

$=$ *RHS*