TRIGONOMETRY (Q 4 & 5, PAPER 2)

2001

4 (a) The length of an arc of a circle is 10 cm. The radius of the circle is 4 cm. The measure of the angle at the centre of the circle subtended by the arc is θ.

(i) Find θ in radians.

(ii) Find θ in degrees, correct to the nearest degree.

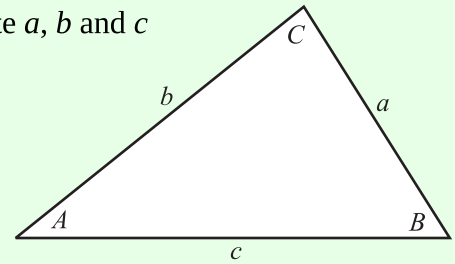
4 (b) (i) Write $\cos 2x$ in terms of $\sin x$.

(ii) Hence, find all the solutions of the equation $\cos 2x - \sin x = 1$ in the domain $0^\circ \le x \le 360^\circ$.

4 (c) A triangle has sides a, b and c. The angles opposite a, b and c are A, B and C, respectively.

(i) Prove that $a^2 = b^2 + c^2 - 2bc\cos A$.

(ii) Show that $c(b\cos A - a\cos B) = b^2 - a^2$.



SOLUTION

4 (a) (i)

$10 = 4\theta \Rightarrow \theta = 2\cdot5$ rad

Arc length s

$s = r\theta$ **6**

4 (a) (ii)

$$2\cdot5 \text{ rad} = 2\cdot5 \times \frac{180^\circ}{\pi} = 143^\circ$$

Degrees to radians: $\times \dfrac{\pi}{180^\circ}$

Radians to degrees: $\times \dfrac{180^\circ}{\pi}$

4 (b) (i)

$\cos 2x = \cos^2 x - \sin^2 x$

$= (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$

$\cos 2A = \cos^2 A - \sin^2 A$ **14**

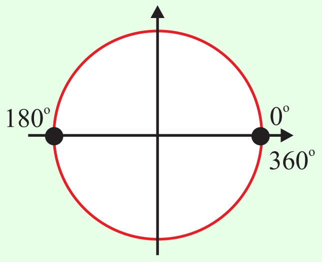
$\cos^2 A + \sin^2 A = 1$ **8**

4 (b) (ii)

$\cos 2x - \sin x = 1 \Rightarrow 1 - 2\sin^2 x - \sin x - 1 = 0$

$\Rightarrow 2\sin^2 x + \sin x = 0 \Rightarrow \sin x(2\sin x + 1) = 0$

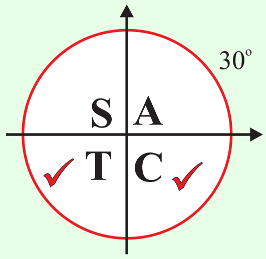
$\Rightarrow \sin x = 0$



$x = 0°, 360°$

$= 180°$

$\Rightarrow \sin x = -\frac{1}{2}$



$x = 210°$ [Third quadrant]

$= 330°$ [Fourth quadrant]

Ans: 0°, 180°, 210°, 330°, 360°

4 (c) (i)

Proof of the Cosine Rule.

PROOF OF THE COSINE RULE

Applying Pythagoras to right-angled triangle **1**:

$a^2 = (c-x)^2 + h^2$

$\Rightarrow a^2 = (h^2 + x^2) + c^2 - 2cx$

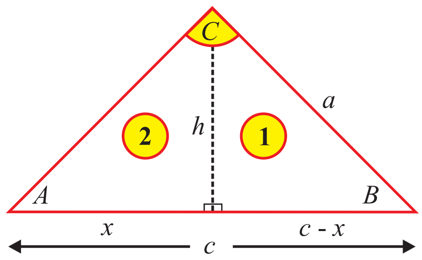
Applying Pythagoras to right-angled triangle **2**:

$b^2 = h^2 + x^2$

$\Rightarrow a^2 = b^2 + c^2 - 2cx$

Also $\cos A = \frac{x}{b} \Rightarrow x = b\cos A$

$\therefore a^2 = b^2 + c^2 - 2bc\cos A$



4 (c) (ii)

Look at the angles. They are *A* and *B*. There is a minus between them. Write down the *A* version of the cosine rule and the *B* version of the cosine rule and subtract them.

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$a^2 - b^2 = b^2 - a^2 - 2bc\cos A + 2ac\cos B$

$\Rightarrow 2c(b\cos A - a\cos B) = 2b^2 - 2a^2$

$\Rightarrow c(b\cos A - a\cos B) = b^2 - a^2$

5 (a) Evaluate $\lim_{\theta \to 0} \frac{\sin 7\theta}{\sin 2\theta}$.

5 (b) xyz is a triangle where $|xy| = 8$ cm and $|yz| = 6$ cm. Given that the area of triangle xyz is 12 cm^2, find

(i) the two possible values of $|\angle xyz|$

(ii) the two possible values of $|xz|$, correct to one decimal place.

5 (c) A is an obtuse angle such that $\sin\left(A + \frac{\pi}{6}\right) + \sin\left(A - \frac{\pi}{6}\right) = \frac{4\sqrt{3}}{5}$.

(i) Find sin A and tan A.

(ii) Given that $\tan(A+B) = \frac{1}{2}$, find tan B and express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbf{Z}$ and $q \neq 0$.

Solution

5 (a)

$$\lim_{\theta \to 0} \frac{\sin 7\theta}{\sin 2\theta} = \lim_{\theta \to 0} \left(\frac{\sin 7\theta}{7\theta} \times \frac{2\theta}{\sin 2\theta} \times \frac{7\theta}{2\theta} \right) = \frac{7}{2}$$

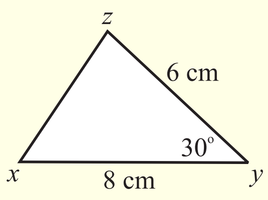
$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ 20

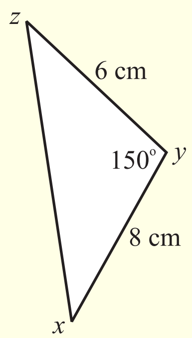
OR

$\lim_{\theta \to 0} \frac{\theta}{\sin \theta} = 1$ 20

5 (b) (i)

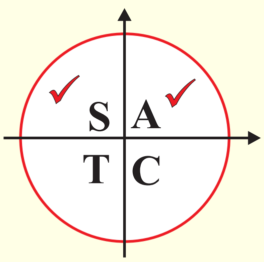
$A = \frac{1}{2} ab \sin C$ 5





$12 = \frac{1}{2}(8)(6)\sin|\angle xyz| \Rightarrow \sin|\angle xyz| = \frac{1}{2}$

Sine is positive in the first and second quadrants. Therefore, there are two possible values for the angle.

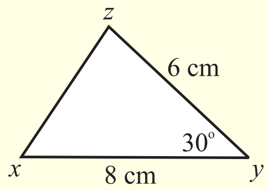


$\angle xyz = 30^\circ$ [First quadrant]

$= 150^\circ$ [Second quadrant]

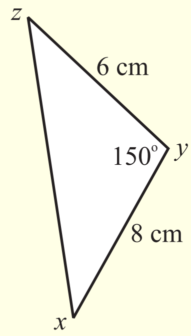
5 (b) (ii)

$a^2 = b^2 + c^2 - 2bc\cos A$ 19



First triangle: $|xz|^2 = 8^2 + 6^2 - 2(8)(6)\cos 30^\circ$

$\Rightarrow |xz| = 4{\cdot}1$ cm



Second triangle: $|xz|^2 = 8^2 + 6^2 - 2(8)(6)\cos 150^\circ$

$\Rightarrow |xz| = 13{\cdot}5$ cm

5 (c)

A is obtuse which means it is in the second quadrant.

5 (c) (i)

Compound Angles	
$\sin(A+B) = \sin A\cos B + \cos A\sin B$	 9
$\sin(A-B) = \sin A\cos B - \cos A\sin B$	 10

$\sin(A+30^\circ) + \sin(A-30^\circ) = \frac{4\sqrt{3}}{5}$

$\Rightarrow \sin A\cos 30^\circ + \sin 30^\circ\cos A + \sin A\cos 30^\circ - \sin 30^\circ\cos A = \frac{4\sqrt{3}}{5}$

$\Rightarrow 2\sin A\cos 30^\circ = \frac{4\sqrt{3}}{5} \Rightarrow 2\sin A \times \frac{\sqrt{3}}{2} = \frac{4\sqrt{3}}{5}$

$\Rightarrow \sin A = \frac{4}{5}$

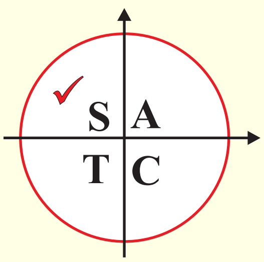
Draw a right-angled triangle and use Pythagoras to find the length of the third side.

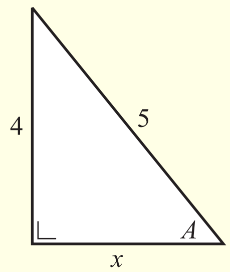
$x^2 + 4^2 = 5^2 \Rightarrow x = 3$

$\sin\theta = \frac{y}{r} = \frac{\text{Opposite}}{\text{Hypotenuse}}$ 2

$\tan\theta = \frac{y}{x} = \frac{\text{Opposite}}{\text{Adjacent}}$ 3

$x^2 + y^2 = r^2$ 4





You can see that tan is negative in the second quadrant.

$\therefore \tan A = -\frac{4}{3}$

5 (c) (ii)

$$\tan(A+B)=\frac{1}{2}\Rightarrow\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{1}{2}$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$ 15

$\Rightarrow 2\tan A+2\tan B=1-\tan A\tan B$

$\Rightarrow 2(-\tfrac{4}{3})+2\tan B=1-(-\tfrac{4}{3})\tan B$

$\Rightarrow -8+6\tan B=3+4\tan B$

$\Rightarrow 2\tan B=11\Rightarrow \tan B=\tfrac{11}{2}$