

TRIGONOMETRY (Q 4 & 5, PAPER 2)

2010

4 (a) The area of a triangle  $PQR$  is  $20 \text{ cm}^2$ .  $|PQ| = 10 \text{ cm}$  and  $|PR| = 8 \text{ cm}$ .

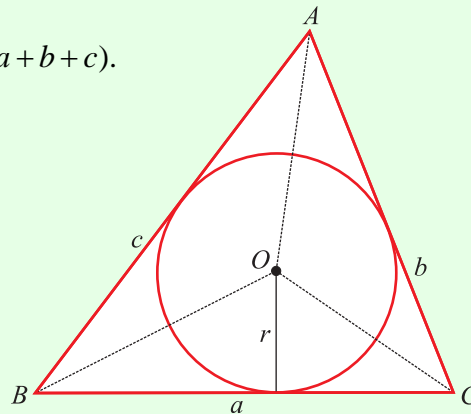
Find the two possible values of  $|\angle QPR|$ .

(b) Find all the solutions of the equation  $\cos 2x = \cos x$  in the domain  $0^\circ \leq x \leq 360^\circ$ .

(c)  $ABC$  is a triangle with sides of lengths  $a$ ,  $b$  and  $c$ , as shown.

Its incircle has centre  $O$  and radius  $r$ .

(i) Show that the area of  $\triangle ABC$  is  $\frac{1}{2}r(a+b+c)$ .

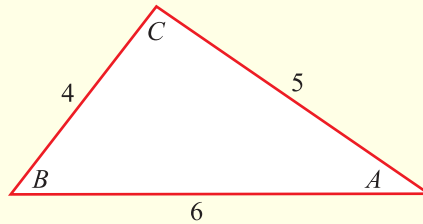


(ii) The lengths of the sides of a triangle are  $a = p^2 + q^2$ ,  $b = p^2 - q^2$ , and  $c = 2pq$ , where  $p$  and  $q$  are natural numbers and  $p > q$ . Show that this triangle is right-angled.

(iii) Show that the radius of the incircle of the triangle in part (ii) is a whole number.

5 (a) Given that  $\tan \theta = \frac{1}{3}$ , show that  $\tan 2\theta = \frac{3}{4}$ .

(b) A triangle has sides of lengths 4, 5 and 6.  
The angles of the triangle are  $A$ ,  $B$  and  $C$ , as in the diagram.



(i) Using the cosine rule, show that  $\cos A + \cos C = \frac{7}{8}$ .

(ii) Show that  $\cos(A + C) = -\frac{9}{16}$ .

(c) (i) Show that  $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 2 + 2\cos(A - B)$ .

(ii) Hence solve the equation  $(\cos 4x + \cos x)^2 + (\sin 4x + \sin x)^2 = 2 + 2\sqrt{3}\sin 3x$   
in the domain  $0^\circ \leq x \leq 360^\circ$ .

**ANSWERS**

4 (a)  $30^\circ, 150^\circ$

(b)  $x = 0^\circ, 120^\circ, 240^\circ, 360^\circ$

5 (c) (ii)  $x = 10^\circ, 70^\circ, 130^\circ, 190^\circ, 250^\circ, 310^\circ$