TRIGONOMETRY (Q 4 & 5, PAPER 2)

2004

4 (a) A is an acute angle such that $\tan A = \frac{15}{17}$.
Without evaluating A, find
(i) $\cos A$
(ii) $\sin 2A$.

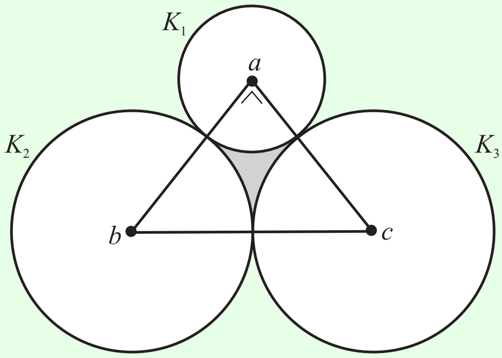
4 (b) (i) Prove that $\cos 2A = \cos^2 A - \sin^2 A$. Deduce that $\cos 2A = 2\cos^2 A - 1$.

(ii) Hence, or otherwise, find the value of θ for which $2\cos\theta - 7\cos(\frac{\theta}{2}) = 0$, where $0^\circ \le \theta \le 360^\circ$. Give your answer correct to the nearest degree.

4 (c) a, b and c are the centres of the circles K_1, K_2 and K_3 repectively. The three circles touch externally and $ab \perp ac$. K_2 and K_3 each have radius $2\sqrt{2}$ cm.

(i) Find, in surd form, the length of the radius of K_1.

(ii) Find the area of the shaded region in terms of π.



5 (a) Prove that $\cos^2 A + \sin^2 A = 1$, where $0^\circ \le A \le 90^\circ$.

5 (b) (i) Show that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2$ simplifies to a constant.

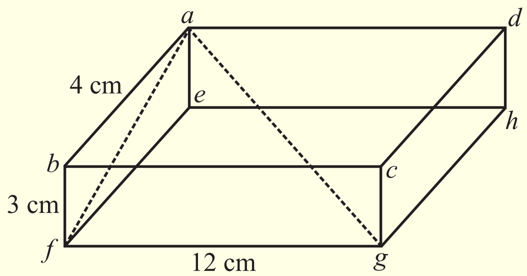
(ii) Express $1 - (\cos x - \sin x)^2$ in the form $a \sin bx$, where $a, b \in \mathbf{Z}$.

5 (c) The diagram shows a rectangular box. Rectangle *abcd* is the top of the box and rectangle *efgh* is the base of the box.
$|ab| = 4$ cm, $|bf| = 3$ cm and $|fg| = 12$ cm.

(i) Find $|af|$.

(ii) Find $|ag|$.

(iii) Find the measure of the acute angle between $[ag]$ and $[df]$. Give your answer correct to the nearest degree.



Answers

4 (a) (i) $\frac{15}{17}$ (ii) $\frac{240}{289}$

4 (b) (ii) 209°

4 (c) (i) $4-2\sqrt{2}$ (ii) $8-8\pi+4\sqrt{2}\pi$

5 (b) (i) 2 (ii) $\sin 2x$

5 (c) (i) 5 cm (ii) 13 cm (iii) 45°