COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

2003

3 (a) Evaluate $(1 \quad -2)\begin{pmatrix} 3 & 0 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

3 (b) (i) Given that $z = 2 - i$, calculate $|z^2 - z + 3|$ where $i^2 = -1$.

(ii) k is a real number such that $\dfrac{-1 + i\sqrt{3}}{-4\sqrt{3} - 4i} = ki$. Find k.

3 (c) $1, \omega, \omega^2$ are the three roots of the equation $z^3 - 1 = 0$.

(i) Prove that $1 + \omega + \omega^2 = 0$.

(ii) Hence, find the value of $(1 - \omega - \omega^2)^5$.

SOLUTION

3 (a)

$$(1 \quad -2)\begin{pmatrix} 3 & 0 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} = (13 \quad -2)\begin{pmatrix} 1 \\ -2 \end{pmatrix} = (17)$$

3 (b) (i)

$$|z^2 - z + 3| = |(2-i)^2 - (2-i) + 3| = |4 - 4i + i^2 - 2 + i + 3|$$

$$= |4 - 3i| = \sqrt{(4)^2 + (-3)^2} = 5$$

3 (b) (ii)

$$\frac{-1 + i\sqrt{3}}{-4\sqrt{3} - 4i} = ki \Rightarrow -1 + i\sqrt{3} = ki(-4\sqrt{3} - 4i)$$

$$\Rightarrow -1 + i\sqrt{3} = 4k - 4k\sqrt{3}i$$

Equating the real parts $\Rightarrow -1 = 4k \Rightarrow k = -\frac{1}{4}$

3 (c) (i)

$$z^3 = 1 \Rightarrow z = (1+0i)^{\frac{1}{3}} = (\cos 2n\pi + i\sin 2n\pi)^{\frac{1}{3}} = \cos\left(\frac{2n\pi}{3}\right) + i\sin\left(\frac{2n\pi}{3}\right)$$

$$n = 0 \Rightarrow z_1 = \cos 0^\circ + i\sin 0^\circ = 1$$

$$n = 1 \Rightarrow z_2 = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) = \cos 120^\circ + i\sin 120^\circ = -\cos 60^\circ + i\sin 60^\circ = -\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}i$$

$$n = 2 \Rightarrow z_3 = \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right) = \cos 240^\circ + i\sin 240^\circ = -\cos 60^\circ - i\sin 60^\circ = -\tfrac{1}{2} - \tfrac{\sqrt{3}}{2}i$$

These 3 roots are called 1, ω, ω^2.

$$\therefore 1 + \omega + \omega^2 = 1 - \tfrac{1}{2} + \tfrac{\sqrt{3}}{2}i - \tfrac{1}{2} - \tfrac{\sqrt{3}}{2}i = 0$$

3 (c) (ii)

$$1 + \omega + \omega^2 = 0 \Rightarrow 1 = -\omega - \omega^2$$

$$\therefore (1 - \omega - \omega^2)^5 = 2^5 = 32$$