SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

LESSON NO. 5: SEQUENCE INEQUALITIES

2005

4 (c) (i) Show that $\frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}}$, where a and b are real numbers.

(ii) The lengths of the sides of a right-angled triangle are a, b and c, where c is the length of the hypotenuse. Using the result from part (**i**), or otherwise, show that $a+b \le c\sqrt{2}$.

SOLUTION

4 (c) (i)

$$\frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}} \Rightarrow \left(\frac{a+b}{2}\right)^2 \le \frac{a^2+b^2}{2}$$

$$\Rightarrow \frac{a^2+2ab+b^2}{4} \le \frac{a^2+b^2}{2} \Rightarrow a^2+2ab+b^2 \le 2a^2+2b^2$$

$$\Rightarrow 0 \le a^2-2ab+b^2 \Rightarrow (a-b)^2 \ge 0$$

4 (c) (ii)

Pythagoras: $a^2+b^2=c^2$

$$\frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}} \Rightarrow \frac{a+b}{2} \le \sqrt{\frac{c^2}{2}} \Rightarrow a+b \le c\sqrt{2}$$

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4 (c) (ii) p, q and r are three numbers in arithmetic sequence. Prove that $p^2+r^2 \ge 2q^2$.

SOLUTION

$p,\ q,\ r \to a,\ a+d,\ a+2d$ [Arithmetic sequence]

$$p^2+r^2 \ge 2q^2 \Rightarrow a^2+(a+2d)^2 \ge 2(a+d)^2$$

$$\Rightarrow a^2+a^2+4ad+4d^2 \ge 2(a^2+2ad+d^2)$$

$$\Rightarrow a^2+a^2+4ad+4d^2 \ge 2a^2+4ad+2d^2$$

$\Rightarrow 2d^2 \ge 0$ [This is always true.]

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4 (c) (ii) a, b, c, d are the first, second, third and fourth terms of a geometric sequence, respectively. Prove that $a^2 - b^2 - c^2 + d^2 \geq 0$.

SOLUTION

$a, b, c, d \to a, ar, ar^2, ar^3$ [Terms of a geometric sequence]

$a^2 - b^2 - c^2 + d^2 \geq 0 \Rightarrow a^2 - a^2r^2 - a^2r^4 + a^2r^6 \geq 0$

$\Rightarrow a^2(1 - r^2 - r^4 + r^6) \geq 0 \Rightarrow 1 - r^2 - r^4 + r^6 \geq 0$

$\Rightarrow 1(1 - r^2) - r^4(1 - r^2) \geq 0 \Rightarrow (1 - r^4)(1 - r^2) \geq 0$

$\Rightarrow (1 - r^2)(1 + r^2)(1 - r^2) \geq 0 \Rightarrow (1 - r^2)^2(1 + r^2) \geq 0$ [This is true for all values of r.]