SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

LESSON NO. 4: SERIES

2001

4 (c) (i) Write $\dfrac{n^3+8}{n+2}$ in the form an^2+bn+c where $a, b, c \in \mathbf{R}$.

(ii) Hence, evaluate $\displaystyle\sum_{n=1}^{30}\frac{n^3+8}{n+2}$.

[Note: $\displaystyle\sum_{n=1}^{k} n=\frac{k}{2}(k+1)$; $\displaystyle\sum_{n=1}^{k} n^2=\frac{k}{6}(k+1)(2k+1)$.]

SOLUTION

4 (c) (i)

$$\frac{n^3+8}{n+2}=\frac{(n)^3+(2)^3}{(n+2)}=\frac{(n+2)(n^2-2n+4)}{(n+2)}=n^2-2n+4$$

Sum of 2 cubes

$a^3+b^3=(a+b)(a^2-ab+b^2)$ **2**

4 (c) (ii)

$\displaystyle\sum_{r=1}^{n} r=S_n=1+2+....+n=\tfrac{n}{2}(n+1)$ **7**

$\displaystyle\sum_{r=1}^{n} r^2=S_n=1^2+2^2+......+n^2=\tfrac{n}{6}(n+1)(2n+1)$ **8**

$$\sum_{n=1}^{30}\frac{n^3+8}{n+2}=\sum_{n=1}^{30}(n^2-2n+4)=\sum_{n=1}^{30}n^2-2\sum_{n=1}^{30}n+4\sum_{n=1}^{30}1$$

$=\tfrac{30}{6}(30+1)(2(30)+1)-2\times\tfrac{30}{2}(30+1)+4\times 30$

$=5(31)(61)-30(31)+120=8,645$

2006

5 (b) (i) Express $\dfrac{2}{(r+1)(r+3)}$ in the form $\dfrac{A}{r+1}+\dfrac{B}{r+3}$.

(ii) Hence find $\displaystyle\sum_{r=1}^{n}\frac{2}{(r+1)(r+3)}$.

(iii) Hence evaluate $\displaystyle\sum_{r=1}^{\infty}\frac{2}{(r+1)(r+3)}$.

SOLUTION

5 (b) (i)

$$\frac{2}{(r+1)(r+3)}=\frac{A}{r+1}+\frac{B}{r+3}\Rightarrow\frac{2}{(r+1)(r+3)}=\frac{A(r+3)+B(r+1)}{(r+1)(r+3)}$$

$\Rightarrow 2=A(r+3)+B(r+1)$

Let $r=-3\Rightarrow 2=-2B\Rightarrow B=-1$

Let $r=-1\Rightarrow 2=2A\Rightarrow A=1$

[This is an identity which means it holds for all values of *r*. Therefore, you any values of *r* you wish. Be clever with your choice.]

$$\therefore\frac{2}{(r+1)(r+3)}=\frac{1}{r+1}-\frac{1}{r+3}$$

5 (b) (ii)

$$\sum_{r=1}^{n}\frac{2}{(r+1)(r+3)}=\sum_{r=1}^{n}\left(\frac{1}{r+1}-\frac{1}{r+3}\right)$$

$$=\frac{1}{2}+\frac{1}{3}-\frac{1}{n+1}-\frac{1}{n+3}$$

$$=\frac{5}{6}-\frac{1}{n+1}-\frac{1}{n+3}$$

SUM TABLE

$r=1$: $\frac{1}{2}-\frac{1}{4}$

$r=2$: $\frac{1}{3}-\frac{1}{5}$

$r=3$: $\frac{1}{4}-\frac{1}{6}$

⋮

$r=n-1$: $\frac{1}{n}-\frac{1}{n+2}$

$r=n$: $\frac{1}{n+1}-\frac{1}{n+3}$

5 (b) (iii)

$$\sum_{r=1}^{\infty}\left(\frac{5}{6}-\frac{1}{r+2}-\frac{1}{r+1}\right)=\frac{5}{6}$$

2004

4 (b) (i) Show that $\dfrac{2}{(2r-1)(2r+1)} = \dfrac{1}{2r-1} - \dfrac{1}{2r+1}$, $r \neq \pm\dfrac{1}{2}$.

(ii) Hence, find $\displaystyle\sum_{r=1}^{n} \frac{2}{(2r-1)(2r+1)}$.

(iii) Evaluate $\displaystyle\sum_{r=1}^{\infty} \frac{2}{(2r-1)(2r+1)}$.

SOLUTION

4 (b) (i)

$$\frac{1}{2r-1} - \frac{1}{2r+1} = \frac{1(2r+1)-1(2r-1)}{(2r-1)(2r+1)} = \frac{2r+1-2r+1}{(2r-1)(2r+1)} = \frac{2}{(2r-1)(2r+1)}$$

4 (b) (ii)

$$\sum_{r=1}^{n} \frac{2}{(2r-1)(2r+1)} = \sum_{r=1}^{n} \left(\frac{1}{2r-1} - \frac{1}{2r+1} \right)$$

$$= 1 - \frac{1}{2n-1}$$

4 (b) (iii)

$$\sum_{r=1}^{\infty} \frac{2}{(2r-1)(2r+1)} = 1$$

SUM TABLE

$r = 1$: $\dfrac{1}{1} - \dfrac{1}{3}$

$r = 2$: $\dfrac{1}{3} - \dfrac{1}{5}$

$r = n-1$: $\dfrac{1}{2n-3} - \dfrac{1}{2n-1}$

$r = n$: $\dfrac{1}{2n-1} - \dfrac{1}{2n+1}$

2002

4 (b) (i) Show that $\frac{2}{k(k+2)}=\frac{1}{k}-\frac{1}{k+2}$, for all $k\in\mathbf{R}$, $k\neq 0, -2$.

(ii) Evaluate, in terms of n, $\sum_{k=1}^{n}\frac{2}{k(k+2)}$.

(iii) Evaluate $\sum_{k=1}^{\infty}\frac{2}{k(k+2)}$.

SOLUTION

4 (b) (i)

$$\frac{1}{k}-\frac{1}{k+2}=\frac{1(k+2)-k}{k(k+2)}=\frac{2}{k(k+2)}$$

4 (b) (ii)

$$\sum_{k=1}^{n}\frac{2}{k(k+2)}=\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+2}\right)$$

$$=1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}$$

$$=\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}$$

4 (b) (iii)

$$\sum_{k=1}^{\infty}\frac{2}{k(k+2)}=\frac{3}{2}$$

SUM TABLE

$k=1$: $\frac{1}{1}-\frac{1}{3}$

$k=2$: $\frac{1}{2}-\frac{1}{4}$

$k=3$: $\frac{1}{3}-\frac{1}{5}$

$k=n-1$: $\frac{1}{n-1}-\frac{1}{n+1}$

$k=n$: $\frac{1}{n}-\frac{1}{n+2}$

2001

4 (b) (i) Show that $\dfrac{1}{(n+2)(n+2)} = \dfrac{1}{n+2} - \dfrac{1}{n+3}$ for $n \in \mathbf{N}$.

(ii) Hence, find $\displaystyle\sum_{n=1}^{k} \frac{1}{(n+2)(n+2)}$ and evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+2)}$.

Solution

4 (b) (i)

$$\frac{1}{n+2} - \frac{1}{n+3} = \frac{1(n+3) - 1(n+2)}{(n+2)(n+3)} = \frac{n+3-n-2}{(n+2)(n+3)} = \frac{1}{(n+2)(n+3)}$$

4 (b) (ii)

$$\sum_{n=1}^{k} \frac{1}{(n+2)(n+3)} = \sum_{n=1}^{k}\left(\frac{1}{n+2} - \frac{1}{n+3}\right)$$

$$= \frac{1}{3} - \frac{1}{k+3}$$

Sum Table

$n = 1$:	$\frac{1}{3} - \frac{1}{4}$
$n = 2$:	$\frac{1}{4} - \frac{1}{5}$
$n = k-1$:	$\frac{1}{k+1} - \frac{1}{k+2}$
$n = k$:	$\frac{1}{k+2} - \frac{1}{k+3}$

4 (b) (ii)

$$\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} = \frac{1}{3}$$