SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

LESSON NO. 2: ARITHMETIC SEQUENCES

2006

4 (a) $-2+2+6+...+(4n-6)$ are the first n terms of an arithmetic series. S_n, the sum of these n terms, is 160. Find the value of n.

SOLUTION

4 (a)

Summing formula: $\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$ 3

$a=-2,\ d=4,\ S_n=160$

$\Rightarrow S_n = \frac{n}{2}[2(-2)+(n-1)(4)]=160$

$\Rightarrow \frac{n}{2}[4n-8]=160 \Rightarrow n(2n-4)=160 \Rightarrow 2n^2-4n-160=0$

$\Rightarrow n^2-2n-80=0 \Rightarrow (n-10)(n+8)=0 \Rightarrow n=10,\ -8$

Answer: n = 10

2003

4 (b) In an arithmetic series, the sum of the second term and the fifth term is 18. The sixth term is greater than the third term by 9.

(i) Find the first term and the common difference.

(ii) What is the smallest value of n such that $S_n > 600$, where S_n is the sum of the first n terms of the series?

SOLUTION

4 (b) (i)

General term: $\boxed{u_n = a+(n-1)d}$ 2

Summing formula: $\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$ 3

The fifty-sixth term of an arithmetic sequence: $u_{56} = a+55d$

$u_2 = a+d$

$u_3 = a+2d$

$u_5 = a+4d$

$u_6 = a+5d$

$u_2+u_5 = 18 \Rightarrow a+d+a+4d = 18 \Rightarrow 2a+5d = 18$....**(1)**

$u_6 = u_3+9 \Rightarrow a+5d = a+2d+9 \Rightarrow 3d = 9 \Rightarrow d = 3$....**(2)**

Substituting the value for d into equation (**2**): $\Rightarrow 2a+5(3)=18 \Rightarrow 2a=3 \Rightarrow a=\frac{3}{2}$

4 (b) (ii)

$S_n = \frac{n}{2}[2a+(n-1)d]=600 \Rightarrow \frac{n}{2}[2(\frac{3}{2})+(n-1)(3)]=600$

$\Rightarrow \frac{n}{2}[3+3n-3]=600 \Rightarrow 3n^2=1200 \Rightarrow n^2=400 \Rightarrow n=20$

The question asks what is the smallest value of n for the sum to exceed 600. 21 terms are needed to exceed this value.

ANSWER: n = 21

2002

4 (c) Three numbers are in arithmetic sequence. Their sum is 27 and their product is 704. Find the three numbers.

SOLUTION

If you are asked to choose three consecutive terms in an arithmetic sequence choose them as: $a-d,\ a,\ a+d$

Call the numbers $a-d,\ a,\ a+d$

Sum: $3a = 27 \Rightarrow a = 9$

Product: $(a-d)a(a+d) = 704 \Rightarrow (9-d)9(9+d) = 704$

$\Rightarrow 81 - d^2 = \frac{704}{9} \Rightarrow d^2 = 81 - \frac{704}{9} = \frac{25}{9} \Rightarrow d = \pm\frac{5}{3}$

Therefore, the 3 numbers are: $a-d,\ a,\ a+d = 9-\frac{5}{3},\ 9,\ 9+\frac{5}{3} = \frac{22}{3},\ 9,\ \frac{32}{3}$

NOTE: There are two values of *d*. Choosing either value gives you the same three numbers in a different order.