SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

LESSON NO. 1: SEQUENCES

2005

4 (b) (ii) A sequence is defined by $u_n = (2-n)2^{n-1}$. Show that $u_{n+2} - 4u_{n+1} + 4u_n = 0$, for all $n \in \mathbf{N}$.

SOLUTION

$u_n = (2-n)2^{n-1} \Rightarrow u_{n+2} = (-n)2^{n+1}$

$u_n = (2-n)2^{n-1} \Rightarrow u_{n+1} = (1-n)2^n$

$\therefore u_{n+2} - 4u_{n+1} + 4u_n = (-n)2^{n+1} - 4(1-n)2^n + 4(2-n)2^{n-1}$

$= 2^{n-1}[(-n)2^2 - 4(1-n)2^1 + 4(2-n)] = 2^{n-1}[-4n - 8 + 8n + 8 - 4n] = 2^{n-1}[0] = 0$

2003

4 (c) (i) $u_1, u_2, u_3, u_4, u_5, ...$ is a sequence where $u_1 = 2$ and $u_{n+1} = (-1)^n u_n + 3$. Evaluate u_2, u_3, u_4, u_5 and u_{10}.

SOLUTION

$u_1 = 2$

$u_2 = (-1)^1 u_1 + 3 = -2 + 3 = 1$

$u_3 = (-1)^2 u_2 + 3 = 1 + 3 = 4$

$u_4 = (-1)^3 u_3 + 3 = -4 + 3 = -1$

$u_5 = (-1)^4 u_4 + 3 = -1 + 3 = 2$ [The sequence starts repeating.]

$2, 1, 4, -1, 2, 1, 4, -1, 2, 1, 4, -1, ...$

As can be seen the tenth term $u_{10} = 1$.

2004

4 (c) (i) The sequence $u_1, u_2, u_3, ...$ is given by $u_{n+1} = \sqrt{4-(u_n)^2}$ and $u_1 = a > 0$. For what value of a will all the terms of the sequence be equal to each other?

SOLUTION

$u_{n+1} = \sqrt{4-(u_n)^2} \Rightarrow u_2 = \sqrt{4-u_1^2}$

$u_1 = a \Rightarrow a = \sqrt{4-a^2}$ [As all the terms are equal $u_1 = u_2 = a$]

$\therefore a^2 = 4 - a^2 \Rightarrow 2a^2 = 4 \Rightarrow a^2 = 2 \Rightarrow a = \sqrt{2}$ [Remember $a > 0$]

2001

4 (a) The sum of the first n terms of an arithmetic series is given by $S_n = 3n^2 - 4n$. Use S_n to find: (i) the first term, u_1

(ii) the sum of the second term and the third term, $u_2 + u_3$.

SOLUTION

4 (a) (i)

$S_n = 3n^2 - 4n \Rightarrow S_1 = u_1 = 3(1)^2 - 4(1) = 3 - 4 = -1$

4 (a) (ii)

$u_2 + u_3 = S_3 - S_1 = 3(3)^2 - 4(3) - (-1)$

$= 27 - 12 + 1 = 16$

$u_n = S_n - S_{n-1}$ 1