Sequences & Series (Q 4 & 5, Paper 1)

Lesson No. 4: Series

2001

4 (c) (i) Write $\dfrac{n^3+8}{n+2}$ in the form an^2+bn+c where $a, b, c \in \mathbf{R}$.

(ii) Hence, evaluate $\displaystyle\sum_{n=1}^{30} \frac{n^3+8}{n+2}$.

[Note: $\displaystyle\sum_{n=1}^{k} n = \frac{k}{2}(k+1)$; $\displaystyle\sum_{n=1}^{k} n^2 = \frac{k}{6}(k+1)(2k+1)$.]

2006

5 (b) (i) Express $\dfrac{2}{(r+1)(r+3)}$ in the form $\dfrac{A}{r+1}+\dfrac{B}{r+3}$.

(ii) Hence find $\displaystyle\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)}$.

(iii) Hence evaluate $\displaystyle\sum_{r=1}^{\infty} \frac{2}{(r+1)(r+3)}$.

2004

4 (b) (i) Show that $\dfrac{2}{(2r-1)(2r+1)} = \dfrac{1}{2r-1} - \dfrac{1}{2r+1}$, $r \neq \pm\dfrac{1}{2}$.

(ii) Hence, find $\displaystyle\sum_{r=1}^{n} \frac{2}{(2r-1)(2r+1)}$.

(iii) Evaluate $\displaystyle\sum_{r=1}^{\infty} \frac{2}{(2r-1)(2r+1)}$.

2002

4 (b) (i) Show that $\dfrac{2}{k(k+2)} = \dfrac{1}{k} - \dfrac{1}{k+2}$, for all $k \in \mathbf{R}$, $k \neq 0, -2$.

(ii) Evaluate, in terms of n, $\displaystyle\sum_{k=1}^{n} \frac{2}{k(k+2)}$.

(iii) Evaluate $\displaystyle\sum_{k=1}^{\infty} \frac{2}{k(k+2)}$.

2001

4 (b) (i) Show that $\dfrac{1}{(n+2)(n+2)} = \dfrac{1}{n+2} - \dfrac{1}{n+3}$ for $n \in \mathbf{N}$.

(ii) Hence, find $\displaystyle\sum_{n=1}^{k} \frac{1}{(n+2)(n+2)}$ and evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+2)}$.

ANSWERS

2001 4 (c) (i) $n^2 - 2n + 4$ (ii) 8,645

2006 5 (b) (i) $\dfrac{1}{r+1} - \dfrac{1}{r+3}$ (ii) $\dfrac{5}{6} - \dfrac{1}{r+2} - \dfrac{1}{r+1}$ (iii) $\dfrac{5}{6}$

2004 4 (b) (ii) $1 - \dfrac{1}{2n+1}$ (iii) 1

2002 4 (b) (ii) $\dfrac{3}{2} - \dfrac{1}{n+1} - \dfrac{1}{n+2}$ (iii) $\dfrac{3}{2}$

2001 4 (b) (ii) $\dfrac{1}{3} - \dfrac{1}{k+3}$; $\dfrac{1}{3}$