SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

2011

4. **(a)** In an arithmetic sequence, the third term is -3 and the sixth term is -15.
Find the first term and the common difference.

(b) Let $u_n = l(\tfrac{1}{2})^n + m(-1)^n$ for all $n \in \mathbb{N}$.

(i) Verify that u_n satisfies the equation $2u_{n+2} + u_{n+1} - u_n = 0$.

(ii) If $a_k = u_k + u_{k+1}$, express a_k in terms of k and l.

(iii) Find $\displaystyle\sum_{k=1}^{\infty} a_k$, in terms of l.

(iv) For $l > 0$, find the least positive integer n for which

$$\sum_{k=1}^{n} a_k > (0.99)\sum_{k=1}^{\infty} a_k.$$

5. **(a)** Find the coefficient of x^8 in the expansion of $(x^2 - 1)^{10}$.

(b) **(i)** Solve the equation:
$$\log_2 x - \log_2 (x-1) = 4\log_4 2.$$

(ii) Solve the equation:
$$3^{2x+1} - 17(3^x) - 6 = 0.$$
Give your answer correct to two decimal places.

(c) Prove by induction that 9 is a factor of $5^{2n+1} + 2^{4n+2}$, for all $n \in \mathbb{N}$.

ANSWERS

4   (a) $a = 5$, $d = -4$

    (b) (ii) $\tfrac{3}{2}l(\tfrac{1}{2})^k$ (iii) $\tfrac{3}{2}l$ (iv) $n = 7$

5   (a) 210

    (b) (i) $x = \tfrac{4}{3}$ (ii) $x = 1.63$