SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

2010

4 (a) Write the recurring decimal 0·474747….. as an infinite geometric series and hence as a fraction.

(b) In an arithmetic sequence, the fifth term is –18 and the tenth term is 12.

(i) Find the first term and the common difference.

(ii) Find the sum of the first fifteen terms of the sequence.

(c) (i) Show that $(r+1)^3-(r-1)^3=6r^2+2$.

(ii) Hence, or otherwise, prove that $\sum_{r=1}^{n} r^2=\frac{n(n+1)(2n+1)}{6}$.

(iii) Find $\sum_{r=11}^{30}(3r^2+1)$.

5 (a) Solve the equation: $\log_2(x+6)-\log_2(x+2)=1$.

(b) Use induction to prove that

$$2+(2\times 3)+(2\times 3^2)+\ldots\ldots\ldots\ldots+(2\times 3^{n-1})=3^n-1,$$

where n is a positive integer.

(c) (i) Expand $\left(x+\frac{1}{x}\right)^2$ and $\left(x+\frac{1}{x}\right)^4$.

(ii) Hence, or otherwise, find the value of $x^4+\frac{1}{x^4}$, given that $x+\frac{1}{x}=3$.

ANSWERS

4 (a) $\frac{47}{99}$

(b) (i) $a=-42$, $d=6$ (ii) $S_{15}=0$

(c) (iii) 27,230

5 (a) $x=2$

(c) (i) $x^2+2+\frac{1}{x^2}$, $x^4+4x^2+6+\frac{4}{x^2}+\frac{1}{x^4}$ (ii) 47