

SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

2009

- 4 (a) Three consecutive terms of an arithmetic series are $4x + 11$, $2x + 11$ and $3x + 17$. Find the value of x .
- (b) (i) Show that $\frac{2}{r^2 - 1} = \frac{1}{r - 1} - \frac{1}{r + 1}$, where $r \neq \pm 1$.
- (ii) Hence, find $\sum_{r=2}^n \frac{2}{r^2 - 1}$.
- (iii) Hence, evaluate $\sum_{r=2}^{\infty} \frac{2}{r^2 - 1}$.
- (c) A finite geometric sequence has first term a and common ratio r . The sequence has $2m + 1$ terms, where $m \in \mathbf{N}$.
- (i) Write down the last term, in terms of a , r , and m .
- (ii) Write down the middle term, in terms of a , r , and m .
- (iii) Show that the product of all the terms of the sequence is equal to the middle term raised to the power of the number of terms.

- 5 (a) Solve for x : $x - 2 = \sqrt{3x - 2}$.
- (b) Prove by induction that, for all positive integers n , 5 is a factor of $n^5 - n$.
- (c) Solve the simultaneous equations
- $$\log_3 x + \log_3 y = 2$$
- $$\log_3(2y - 3) - 2\log_9 x = 1.$$

ANSWERS

- 4 (a) $x = -2$
- (b) (ii) $\frac{3}{2} - \frac{1}{n} - \frac{1}{n+1}$ (iii) $\frac{3}{2}$
- (c) (i) ar^{2m} (ii) ar^m
- 5 (a) $x = 6$
- (c) $x = 2, y = \frac{9}{2}$