SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

2008

4 (a) $2+\frac{2}{3}+\frac{2}{9}+......$ is a geometric series.
Find the sum to infinity of the series.

(b) Given that $u_n = 2(-\frac{1}{2})^n - 2$ for all $n \in \mathbf{N}$,

(i) write down u_{n+1} and u_{n+2}

(ii) show that $2u_{n+2} - u_{n+1} - u_n = 0$.

(c) (i) Write down an expression in n for the sum 1 + 2 + 3 +.........+ n
and an expression in n for the sum $1^2+2^2+3^2+......+n^2$.

(ii) Find, in terms of n, the sum $\sum_{r=1}^{n}(6r^2+2r+5+2^r)$.

5 (a) Find the range of values of x which satisfy the inequality

$$x^2-3x-10 \le 0.$$

(b) (i) Solve the equation

$$2^{x^2} = 8^{2x+9}.$$

(ii) Solve the equation

$$\log_e(2x+3)+\log_e(x-2) = 2\log_e(x+4).$$

(c) Show that there are no natural numbers n and r for which

$\binom{n}{r-1}$, $\binom{n}{r}$ and $\binom{n}{r+1}$ are consecutive terms in a geometric sequence.

ANSWERS

4 (a) 3

(b) (i) $u_{n+1} = 2(-\frac{1}{2})^{n+1} - 2,\ u_{n+2} = 2(-\frac{1}{2})^{n+2} - 2$

(c) (i) $\sum n = \frac{n}{2}(n+1),\ \sum n^2 = \frac{n}{6}(n+1)(2n+1)$

(ii) $n(n+1)(2n+1)+n(n+1)+5n-2+2^{n+1}$

5 (a) $-2 \le x \le 5$

(b) (i) $x=-3, 9$ (ii) $x = 11$