# SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

## 2007

4 (a) Show that $\binom{n}{1}+\binom{n}{2}=\binom{n+1}{2}$ for all natural numbers $n \geq 2$.

(b) $u_1 = 5$ and $u_{n+1} = \frac{n}{n+1}u_n$ for all $n \geq 1, n \in \mathbf{N}$.

(i) Write down the value of each of $u_2$, $u_3$, and $u_4$.

(ii) Hence, by inspection, write an expression for $u_n$ in terms of $n$.

(iii) Use induction to justify your answer for part **(ii)**.

(c) The sum of the first $n$ terms of a series is given by $S_n = n^2 \log_e 3$.

(i) Find the $n^{\text{th}}$ term and prove that the series is arithmetic.

(ii) How many of the terms of the series are less than $12\log_e 27$?

5 (a) Plot, on the number line, the values of $x$ that satisfy the inequality $|x+1| \leq 2$, where $x \in \mathbf{Z}$.

(b) In the expansion of $\left(2x - \frac{1}{x^2}\right)^9$,

(i) find the general term

(ii) find the value of the term independent of $x$.

(c) The $n^{\text{th}}$ term of a series is given by $nx^n$, where $|x| < 1$.

(i) Find an expression for $S_n$, the sum of the first $n$ terms of the series.

(ii) Hence, find the sum to infinity of the series.

**ANSWERS**

4 (b) (i) $\frac{5}{2}, \frac{5}{3}, \frac{5}{4}$ (ii) $u_n = \frac{5}{n}$

(c) (i) $u_n = (2n-1)\ln 3$ (ii) 17

5 (b) (i) $\binom{9}{r}(2x)^{9-r}\left(-\frac{1}{x}\right)^r$ (ii) $-5376$

(c) (i) $S_n = \frac{x(1-x^n)}{(1-x)^2} + \frac{nx^{n+1}}{(1-x)}$

(ii) $S_\infty = \frac{x}{(1-x)^2}$