Sequences & Series (Q 4 & 5, Paper 1)

2006

4 (a) $-2+2+6+...+(4n-6)$ are the first n terms of an arithmetic series. S_n, the sum of these n terms, is 160. Find the value of n.

4 (b) The sum to infinity of a geometric series is $\frac{9}{2}$. The second term of the series is -2. Find the vaue of r, the common ratio of the series.

4 (c) The sequence $u_1, u_2, u_3, ...,$ defined by $u_1 = 3$ and $u_{n+1} = 2u_n + 3$, is as follows: 3, 9, 21, 45, 93, ...

(i) Find u_6, and verify that it is equal to the sum of the first six terms of a geometric series with first term 3 and common ratio 2.

(ii) Given that, for all k, u_k is the sum of the first k terms of a geometric series with first term 3 and common ratio 2, find $\sum_{k=1}^{n} u_k$.

5 (a) Find the value of the middle term of the binomial expansion of $\left(\frac{x}{y}-\frac{y}{x}\right)^8$.

5 (b) (i) Express $\frac{2}{(r+1)(r+3)}$ in the form $\frac{A}{r+1}+\frac{B}{r+3}$.

(ii) Hence find $\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)}$.

(iii) Hence evaluate $\sum_{r=1}^{\infty} \frac{2}{(r+1)(r+3)}$.

5 (c) (i) Given two real numbers a and b, where $a > 1$ and $b > 1$, prove that

$$\frac{1}{\log_b a}+\frac{1}{\log_a b} \geq 2.$$

(ii) Under what condition is $\frac{1}{\log_b a}+\frac{1}{\log_a b} = 2$.

Answers

4 (a) $n = 10$

4 (b) $r = -\frac{1}{3}$

4 (c) (i) $u_6 = 189$ (ii) $\sum_{k=1}^{n} u_k = 6(2^n - 1) - 3n$

5 (a) 70

5 (b) (i) $\dfrac{1}{r+1} - \dfrac{1}{r+3}$ (ii) $\dfrac{5}{6} - \dfrac{1}{r+2} - \dfrac{1}{r+1}$ (iii) $\dfrac{5}{6}$

5 (c) (ii) $a = b$