SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

2005

4 (a) Write the recurring decimal 0·636363.... as an infinite geometric series and hence as a fraction.

4 (b) (i) The first three terms in the binomial expansion of $(1+kx)^n$ are $1-21x+189x^2$. Find the value of *n* and the value of *k*.

(ii) A sequence is defined by $u_n=(2-n)2^{n-1}$. Show that $u_{n+2}-4u_{n+1}+4u_n=0$, for all $n\in\mathbf{N}$.

4 (c) (i) Show that $\dfrac{a+b}{2}\le\sqrt{\dfrac{a^2+b^2}{2}}$, where *a* and *b* are real numbers.

(ii) The lengths of the sides of a right-angled triangle are *a*, *b* and *c*, where *c* is the length of the hypotenuse. Using the result from part (**i**), or otherwise, show that $a+b\le c\sqrt{2}$.

5 (a) Solve for *x*: $\sqrt{10-x}=4-x$.

5 (b) Prove by induction that $\sum_{r=1}^{n}(3r-2)=\frac{n}{2}(3n-1)$.

5 (c) (i) Show that $\dfrac{1}{\log_a b}=\log_b a$, where $a, b>0$ and $a, b\ne 1$.

(ii) Show that $\dfrac{1}{\log_2 c}+\dfrac{1}{\log_3 c}+\dfrac{1}{\log_4 c}+.......+\dfrac{1}{\log_r c}=\dfrac{1}{\log_{r!} c}$, where $c>0, c\ne 1$.

ANSWERS

4 (a) $\frac{7}{11}$

4 (b) (i) $n=7,\ k=-3$

5 (a) $x=1$