SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

2004

4 (a) Show that $3\binom{n}{3}=n\binom{n-1}{2}$ for all natural numbers $n \geq 3$.

4 (b) (i) Show that $\dfrac{2}{(2r-1)(2r+1)}=\dfrac{1}{2r-1}-\dfrac{1}{2r+1}$, $r \neq \pm\dfrac{1}{2}$.

(ii) Hence, find $\displaystyle\sum_{r=1}^{n}\frac{2}{(2r-1)(2r+1)}$.

(iii) Evaluate $\displaystyle\sum_{r=1}^{\infty}\frac{2}{(2r-1)(2r+1)}$.

4 (c) (i) The sequence $u_1, u_2, u_3, \ldots$ is given by $u_{n+1}=\sqrt{4-(u_n)^2}$ and $u_1=a>0$. For what value of a will all the terms of the sequence be equal to each other?

(ii) p, q and r are three numbers in arithmetic sequence. Prove that $p^2+r^2 \geq 2q^2$.

5 (a) Find the fifth term in the expansion of $\left(x^2-\dfrac{1}{x}\right)^6$ and show that it is independent of x.

5 (b) (i) In a geometric series, the second term is 8 and the fifth term is 27. Find the first term and the common ratio.

(ii) Solve $\log_4 x-\log_4(x-2)=\frac{1}{2}$.

5 (c) Prove by induction that $2^n \geq n^2$, $n \in \mathbf{N}$, $n \geq 4$.

ANSWERS

4 (b) (ii) $1-\dfrac{1}{2n+1}$ (iii) 1

4 (c) (i) $a=\sqrt{2}$

5 (a) 15

5 (b) (i) $a=\frac{16}{3}$, $r=\frac{3}{2}$ (ii) $x=4$