SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

2003

4 (a) Express the recurring decimal 0.252525... in the form $\frac{p}{q}$ where $p, q \in \mathbf{N}$ and $q \neq 0$.

4 (b) In an arithmetic series, the sum of the second term and the fifth term is 18. The sixth term is greater than the third term by 9.

(i) Find the first term and the common difference.

(ii) What is the smallest value of *n* such that $S_n > 600$, where S_n is the sum of the first *n* terms of the series?

4 (c) (i) $u_1, u_2, u_3, u_4, u_5, ...$ is a sequence where $u_1 = 2$ and $u_{n+1} = (-1)^n u_n + 3$. Evaluate u_2, u_3, u_4, u_5 and u_{10}.

(ii) *a*, *b*, *c*, *d* are the first, second, third and fourth terms of a geometric sequence, respectively. Prove that $a^2 - b^2 - c^2 + d^2 \geq 0$.

5 (a) Solve for *x*: $x = \sqrt{7x-6} + 2$.

5 (b) Use induction to prove that 8 is a factor of $7^{2n+1} + 1$ for any positive integer *n*.

5 (c) Consider the binomial expansion of $\left(ax + \frac{1}{bx}\right)^8$, where *a* and *b* are non-zero real numbers.

(i) Write down the general term.

(ii) Given that the coefficient of x^2 is the equal to the coefficient of x^4, show that $ab = 2$.

ANSWERS

4 (a) $\frac{25}{99}$

4 (b) (i) $a = \frac{3}{2}, d = 3$ (ii) $n = 21$

4 (c) (i) $u_2 = 1, u_3 = 4, u_4 = -1, u_5 = 2, u_{10} = 1$

5 (a) $x = 10$

5 (c) (i) $\binom{8}{r}(ax)^{8-r}\left(\frac{1}{bx}\right)^r$