Sequences & Series (Q 4 & 5, Paper 1)

2002

4 (a) Find in terms of n, the sum of the first n terms of the geometric series $3+\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+...$

4 (b) (i) Show that $\frac{2}{k(k+2)}=\frac{1}{k}-\frac{1}{k+2}$, for all $k \in \mathbf{R}$, $k \neq 0, -2$.

(ii) Evaluate, in terms of n, $\sum_{k=1}^{n}\frac{2}{k(k+2)}$.

(iii) Evaluate $\sum_{k=1}^{\infty}\frac{2}{k(k+2)}$.

4 (c) Three numbers are in arithmetic sequence. Their sum is 27 and their product is 704. Find the three numbers.

5 (a) Find the value of x in each case:

(i) $\frac{8}{2^x}=32$

(ii) $\log_9 x=\frac{3}{2}$.

5 (b) The first three terms in the binomial expansion of $(1+ax)^n$ are $1+2x+\frac{7}{4}x^4$.

(i) Find the value of a and the value of n.

(ii) Hence, find the middle term in the expansion.

5 (c) Prove by induction that, for any positive integer n, $x+x^2+x^3+...+x^n=\frac{x(x^n-1)}{x-1}$,

where $x \neq 1$.

Answers

4 (a) $6\left[1-(\frac{1}{2})^n\right]$

4 (b) (ii) $\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}$ (iii) $\frac{3}{2}$

4 (c) $\frac{32}{3}$, 9, $\frac{22}{3}$

5 (a) (i) $x=-2$ (ii) $x=27$

5 (b) (i) $a=\frac{1}{4}$, $n=8$ (ii) $\frac{35x^4}{128}$