SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

2001

4 (a) The sum of the first n terms of an arithmetic series is given by $S_n = 3n^2 - 4n$. Use S_n to find: (i) the first term, u_1

(ii) the sum of the second term and the third term, $u_2 + u_3$.

4 (b) (i) Show that $\dfrac{1}{(n+2)(n+2)} = \dfrac{1}{n+2} - \dfrac{1}{n+3}$ for $n \in \mathbf{N}$.

(ii) Hence, find $\displaystyle\sum_{n=1}^{k} \frac{1}{(n+2)(n+2)}$ and evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+2)}$.

4 (c) (i) Write $\dfrac{n^3+8}{n+2}$ in the form $an^2 + bn + c$ where $a, b, c \in \mathbf{R}$.

(ii) Hence, evaluate $\displaystyle\sum_{n=1}^{30} \frac{n^3+8}{n+2}$.

[Note: $\displaystyle\sum_{n=1}^{k} n = \frac{k}{2}(k+1)$; $\displaystyle\sum_{n=1}^{k} n^2 = \frac{k}{6}(k+1)(2k+1)$.]

5 (a) The second term, u_2, of a geometric sequence is 21. The third term, u_3, is -63. Find

(i) the common ratio

(ii) the first term.

5 (b) (i) Solve $\log_6(x+5) = 2 - \log_6 x$ for $x > 0$.

(ii) In the binomial expansion of $(1+kx)^6$, the coefficient of x^4 is 240. Find the two possible values of k.

5 (c) Use induction to prove that, for n a positive integer, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for all $\theta \in \mathbf{R}$ and $i^2 = -1$.

ANSWERS

4 (a) (i) -1 (ii) 16

4 (b) (ii) $\dfrac{1}{3} - \dfrac{1}{k+3}$; $\dfrac{1}{3}$

4 (c) (i) $n^2 - 2n + 4$ (ii) 8,645

5 (a) (i) $r = -3$ (ii) $a = -7$

5 (b) (i) $x = 4$ (ii) $k = \pm 2$