SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

2000

4 (a) The first three terms of a geometric sequence are

$$2x-4, x+1, x-3.$$

Find the two possible values of x.

4 (b) Given that $u_n = \frac{1}{2}(4^n - 2^n)$ for all integers n, show that

$$u_{n+1} = 2u_n + 4^n.$$

4 (c) (i) Given that $g(x) = 1 + 2x + 3x^2 + 4x^3 \ldots$ where $-1 < x < 1$, show that

$$g(x) = \frac{1}{(1-x)^2}.$$

(ii) $P(n) = u_1 u_2 u_3 u_4 \ldots u_n$ where $u_k = ar^{k-1}$ for $k = 1, 2, 3, \ldots, n$ and $a, r \in \mathbf{R}$.

Write $P(n)$ in the form $a^n r^{f(n)}$ where $f(n)$ is a quadratic expression in n.

5 (a) Express the recurring decimal $1.\dot{2}$ in the form $\frac{a}{b}$ where $a, b \in \mathbf{N}$.

5 (b) Prove by induction that $n! > 2^n$, $n \in \mathbf{N}$, $n \geq 4$.

5 (c) (i) Solve for x

$$2\log_9 x = \frac{1}{2} + \log_9(5x+18),\ x > 0.$$

(ii) Solve for x

$$3e^x - 7 + 2e^{-x} = 0.$$

ANSWERS

4 (a) 1, 11

4 (c) (ii) $a^n \times r^{\frac{n}{2}(n-1)}$

5 (a) $\frac{11}{9}$

5 (c) (i) 6, 9 (ii) $-\ln 3$, $\ln 2$