SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

1999

4 (a) Solve $\binom{n+4}{2}=91$, for $n \in \mathbf{N}$.

4 (b) (i) The nth term of an arithmetic series is $3n+2$.
Find S_n, the sum of the first n terms, in terms of n.

(ii) Evaluate, in terms of n, $\displaystyle\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+1}\right)$.

4 (c) Let $f(x)=\displaystyle\sum_{n=1}^{\infty} q^{n-1}x^n$, where $|x|<1$ and $0<q<1$.

Show that $f(x)=\dfrac{x}{1-qx}$.

If $g(x)=\dfrac{1}{1-(1-q)f(x)}$, show that $g(x)=\dfrac{1-qx}{1-x}$.

5 (a) Find the coefficient of a^3 in $(2+a)^5$.

5 (b) (i) Solve the equation $\sqrt{2x+7}=2+\sqrt{x}$.

(ii) If $x>0$ and $x \neq 1$, show that

$$\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+\frac{1}{\log_5 x}=\frac{1}{\log_{30} x}.$$

Note: $\log_b a=\dfrac{\log_c a}{\log_c b}$.

5 (c) Prove by induction that $\displaystyle\sum_{r=1}^{n} r^2=\tfrac{n}{6}(n+1)(2n+1)$.

ANSWERS

4 (a) 10

(b) (i) $\frac{n}{2}(3n+7)$ (ii) $1-\frac{1}{n+1}$

5 (a) 40

5 (b) (i) 1, 9