SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

1998

4 (a) Find the sum to infinity of the geometric series

$1+(\tfrac{2}{3})+(\tfrac{2}{3})^2+(\tfrac{2}{3})^3+\ldots$

(b) If for all integers n,

$u_n = 3+n(n-1)^2,$

show that

$u_{n+1}-u_n = 3n^2-n.$

(c) Show that for n a natural number $\dfrac{1}{4n^2-1}=\dfrac{1}{2}\left(\dfrac{1}{2n-1}-\dfrac{1}{2n+1}\right)$

Let $u_n = \dfrac{1}{4n^2-1}$.

Find $\displaystyle\sum_{n=1}^{\infty} u_n$.

Find the least value of r such that

$\displaystyle\sum_{n=1}^{r} u_n > \frac{99}{100}\sum_{n=1}^{\infty} u_n, r \in \mathbf{N}.$

5 (a) Find the value of the term which is independent of x in the expansion of

$\left(x^2-\dfrac{1}{x}\right)^9.$

(b) Solve

$\log_5(x-2)=1-\log_5(x-6),\ x\in\mathbf{R}, x>6.$

(c) Let $u_n=(1+x)^n-1-nx$ for $n\in\mathbf{N}_0, x\in\mathbf{R}$ and $x>-1$ and where $u_n=u_n(x)$.

Show that

$u_{n+1}\geq u_n$

(i) when $x = 0$

(ii) when $x > 0$

(iii) when $-1<x<0$.

Show that $u_2 \geq 0$.

Hence, or otherwise, deduce that

$(1+x)^n \geq 1+nx,\ x>-1.$

ANSWERS

4 (a) 3

(b) $S_\infty = \frac{1}{2}$; $r \geq 50$

5 (a) 84

(b) $x = 7$