SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

1997

4 (a) Write down, or find, in terms of n, the sum of n terms of the infinite arithmetic series

$$1 + 2 + 3 + ... + n.$$

(b) If for all integers n,

$u_n = (5n-3)2^n$,

verify that

$u_{n+1} - 2u_n = 5(2^{n+1})$.

(c) Consider the sum to n terms, S_n, of the following finite geometric series

$S_n = 1 + (1+x) + (1+x)^2 + (1+x)^3 + ... + (1+x)^{n-1}$

for $x > 0$.

Show that the coefficient of x^2 in the above expression for S_n is

$\binom{2}{2} + \binom{3}{2} + \binom{4}{2} + ... \binom{n-1}{2}$.

By finding S_{25} in terms of x and by considering the coefficient of x^2 in S_{25}, find the value of p and the value of q for which

$\binom{2}{2} + \binom{3}{2} + \binom{4}{2} + ... \binom{24}{2} = \binom{p}{q}$, where $p, q \in \mathbf{N}$.

5 (a) Solve

$\log_5 x = 1 + \log_5 \left(\frac{3}{2x-1} \right), x \in \mathbf{R}, x > \frac{1}{2}$.

(b) (i) Solve $\frac{x-3}{x-4} < -2,\ x \neq 4,\ x \in \mathbf{R}$.

(ii) If k is a positive integer and 720 is the coefficient of x^3 in the binomial expansion of $(k+2x)^5$, find the value of k.

(c) Prove by induction that 8 is a factor of $3^{2n} - 1$ for $n \in \mathbf{N}_0$.

ANSWERS

4 (a) $\frac{n(n+1)}{2}$

(c) $p = 25$, $q = 3$

5 (a) 3

(b) (i) $\frac{5}{3} < x < 4$ (ii) $k = 3$