SEQUENCES & SERIES (Q 4 & 5, PAPER 1)

1996

4 (a) Find S_n, the sum of n terms, of the geometric series

$$2+\frac{2}{3}+\frac{2}{3^2}+\dots\dots\dots\dots+\frac{2}{3^{n-1}}.$$

If $S_n=\frac{242}{81}$, find the value of n.

(b) (i) Show that $\frac{1}{\sqrt{n+1}+\sqrt{n}}$ is equal to $\sqrt{n+1}-\sqrt{n}$.

(ii) If $u_n=\frac{1}{\sqrt{n+1}+\sqrt{n}}$ find an expression for the sum of the first n terms in terms of n.

(c) $u_1, u_2, u_3, \dots\dots\dots u_n$ is a sequence, where $u_n=1+2+3+\dots\dots+n$.

(i) Show $u_n=\frac{n}{2}(n+1)$.

(ii) Express u_n-u_{n-1} in terms of n.

(iii) Show $u_n+u_{n-1}=n^2$.

(iv) Find $u_n^{\ 2}-u_{n-1}^{\ 2}$.

Hence, show that $\sum_1^n (u_n^{\ 2}-u_{n-1}^{\ 2})=1+2^3+3^3+\dots\dots+n^3$ where $u_0=0$.

5 (a) Solve the simultaneous equations

$\log(x+y)=2\log x$

$\log y=\log 2+\log(x-1)$ where $x>1, y>0$.

(b) (i) Write the binomial expansion of $(a+b)^4$ in ascending powers of b.

Find $\left(x+\frac{1}{x}\right)^4-\left(x-\frac{1}{x}\right)^4$ in its simplest form.

(ii) Write u_{r+1}, the general term of the binomial expansion of $(3+2x)^n$ in terms of x, r and n.
If the coefficients of x^5 and x^6 are equal, find the value of n.

(c) Prove by induction that if $y=x^n$, then $\frac{dy}{dx}=nx^{n-1}$, $n\in\mathbf{N}_0$.

ANSWERS

4 (a) $S_n = 3\left(1 - \frac{1}{3^n}\right)$, $n = 5$

(b) (ii) $\sqrt{n+1} - 1$

(c) (ii) n (iv) n^3

5 (a) $x = 2$, $y = 2$

(b) (i) $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$, $8x^2 + \frac{8}{x^2}$

(ii) $u_{r+1} = \binom{n}{r} 3^{n-r} (2x)^r$, $n = 14$